CAIM 2025

The 32nd International Conference on Applied and Industrial Mathematics

September 18-21, 2025

Bucharest, Romania

e-Book of Abstracts

BUCHAREST
2025

CAIM 2025

September 18-21, 2025
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Book of Abstracts

Organized by

National University of Science and Technology POLITEHNICA Bucharest

Faculty of Applied Sciences

ROMAI

"Gh. Mihoc- C. Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy

Volume Editor: Stelian Ion

Printed by PIM Publishing House
Address: 11, Ştefan cel Mare Street, Iaşi, Romania
e-mail: editura@pimcopy.ro

ISSN 2537-2688

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Plenary talks

CAIM 2025, Bucharest, September 18–21, 2025

Flux – Flow Constructal Couplings in the Electropermeabilization of the Cell Membrane

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Through electromagnetic field analysis and numerical experiments, the influence of the applied electromagnetic field (EMF) frequency on the electropermeabilization (EP) conditions is studied. The physical quantity correlated with EP triggering is the transmembrane voltage induced before the pore generation process begins. The Constructal Law (CL) of physics, which emphasizes the freedom of reconfiguration as a prerequisite for evolution, persistence, and survival of systems, manifests here through the intrinsic couplings of the associated electromagnetic field. The "morphology" of the system – what flows (EMF) and in what it flows (substance) – must change (naturally) or be changed (by design) so that the work required by the flows is minimal, the heat produced is transferred to the outside as efficiently as possible, etc. In this EP numerical experiment, the total electric current comprises the conduction current (which flows through highly conductive or high-electrical conductivity media at any frequency) and the displacement current (which flows through highly polarizable or high-permeability media at higher frequencies). The conduction current dominates the low-frequency range: it flows outside the cells (does not cross the membrane). The displacement current becomes more critical at higher frequencies. The role interplay between these currents accompanies the EP. The EMF intrinsically couples electric and magnetic fields, which source each other. Evolution's constructal nature applies to EMFs, governing their action-reaction morphing into propagation.

Numerical Problems in Aerospace: Regression, Optimization, Control, ODE integrator

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The national aerospace community experienced in the last two decades the opportunities and challenges brought by the European Space Agency and European Commission funded projects where old and new problems can be equally approached for various technical readiness levels. The presentation includes more topics that are described, with original solutions, from an engineering standpoint.

• Supersonic Wind Tunnel operation requires an improvement in forces measurement data reduction method, from moving window linear regression to polygonal line with controlled diffusion and third/fifth order regression splines that are developed in an original formulation, all three models being presented.

• Optimization of operation for blow-down supersonic wind tunnels requires a low-fidelity Digital Twin (DT) that must be fast and can be easily recalibrated. Therefore, the tank discharge problem as an interim solution to a developed DT is solved with bang-bang type of controller regulating both the total pressure and massflow, enabling the determination of the useful run-time of the

facility for a given flow regime.
• The startup process of the facility brings a dynamic load on the model's beam support. The Euler-Bernouli equation applied for the beam is integrated with a time explicit finite difference solver, that will be part of the DT.
• Supersonic ramp as planar intake is subject of optimization considering multiple ramp angles for a given Mach number. Standard interior point method is used for the problem that can be handled without constraints. A family of optimized configurations are compared with published results. Calorically perfect gas and thermally perfect gas models are compared. -A Single Stage to Orbit optimization is formulated, considering the minimal vehicle mass and the thrust profile to enable direct circular orbit. Differential Evolution optimizer is used with a penalty function. Solution uniqueness is discussed.
• A Vertical Takeoff and Landing vehicle vertical trajectory is optimized as notional trajectory, considering two constant engine firings only. Interior point optimizer is used with moderate repeatability. Solution uniqueness is discussed.
• An ODE integrator is based on natural splines, as corrector step. Based on intuition, it is hand derived and can have a certain future is studied more in-depth. An important application is the reentry of space vehicles, debris or natural bodies, where an advanced standard atmosphere model can take half of the computational time. The mentioned problems are presented and discussed.

Homogenization Results for Diffusion Problems in Composites with General Imperfect Contact Conditions

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During the last two decades, the study of the effective properties of heterogeneous composite materials with imperfect contact between their constituents has been a topic of huge interest for a broad category of researchers, such as engineers, mathematicians, biologists, and physicists. It is well-known that the macroscopic properties of such materials are strongly affected by the imperfect bonding between their constitutive components. In this talk, we shall present some homogenization results for a stationary heat diffusion problem set in a periodic two-phase composite material with imperfect contact conditions between its building blocks. On the interface separating the two phases, we prescribe non-standard transmission conditions of non-local type, obtained via a concentration procedure, by supposing that the interface is the limit of a thin anisotropic layer (see [1]). The homogenization process is carried out by using the periodic unfolding method (see [2]) and, as a result of such a procedure, several new macroscopic models are obtained at the limit (see [3, 4, 5]).
This contribution is based on recent joint work with Micol Amar and Daniele Andreucci (Sapienza University of Rome, Italy).

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Mathematical Methods for the Study of Mechanical and Physical Phenomena

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This work explores advanced applications of distribution theory and fractional calculus to a range of problems in mechanics, electrodynamics, and applied mathematics. In the first part, distribution theory is employed to address complex mechanical systems, including generalized solutions to Cauchy's problem for longitudinal vibrations of elastic rods, a unified approach for solving bending problems in elastic rods (with and without elastic foundations), and the quasi-static analysis of vibrations in bars with discontinuities. Further, Dirac sequences are characterized with specific applications in electrodynamics, and dimensional analysis is extended to generalized functions in the distribution space. The methodology is also applied to longitudinal vibrations in viscoelastic rods of Maxwell type, offering new perspectives on mathematical modeling and boundary-value problem resolution.

In the second part, fractional calculus is investigated from both theoretical and computational perspectives. Fundamental concepts and formulae are reviewed, followed by an assessment of numerical accuracy in fractional computations. Applications include numerical solutions of Fredholm–Volterra integro-differential equations, Riccati fractional differential equations, and two-dimensional fractional-order PDEs using hybrid functions. The study extends to symmetry analysis in nonconservative field theories on time scales, applications to RLC circuits, structural health monitoring through modal parameter analysis, and Hamilton–Pfaff type PDEs via multidimensional fractional optimization. Additional interdisciplinary applications are discussed, including fractional models in epidemiology—highlighting the COVID-19 pandemic—and connections with chaos theory. The unified treatment of distribution theory and fractional calculus presented here provides a versatile analytical framework for tackling challenging problems in modern science and engineering.

Introduction

The present work investigates the interplay between distribution theory and fractional calculus, with emphasis on their applications in mechanics, electrodynamics, and multidisciplinary physical models. Distribution theory, by extending classical analysis to encompass generalized functions,

offers a powerful mathematical framework for modeling and solving problems involving singularities, discontinuities, and concentrated sources. Fractional calculus, which generalizes the concept of differentiation and integration to arbitrary non-integer orders, provides an equally powerful tool for capturing memory effects and hereditary properties in diverse physical systems. Together, these two domains form a complementary set of methods for tackling challenging mathematical models encountered in modern science and engineering.

Distribution Theory in Mechanics

The first part of this study focuses on the application of distribution theory to mechanics, particularly in the analysis of elastic bars and rods. The governing equation for longitudinal vibrations of an elastic rod can be expressed in classical form as $\frac{\partial^2 u(x,t)}{\partial t^2} - c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = f(x,t)$, where $u(x,t)$ is the longitudinal displacement, c is the wave speed, and $f(x,t)$ is the external force density. When f contains impulsive terms (e.g., Dirac delta $\delta(x - x_0)$), the solution must be interpreted in the distributional sense, allowing $f(x,t) = F_0 \delta(x - x_0) H(t)$, denoting by $H(t)$, $t \in \mathbb{R}$ the Heaviside's function and $\delta(x) \in D'_+ \subset D'(\mathbb{R})$ Dirac's distribution.

Using the fiber generalized contractions theorem, a unified method is developed for the resolution of boundary value problems, including the Cauchy problem, the bending of elastic rods on elastic foundations, and quasi-static vibrations of bars with discontinuities. This approach yields generalized solutions valid beyond the limits of classical differentiability.

The generalized equation in displacement of the longitudinal vibrations of elastic rods is $L(\widetilde{\partial}_x, \widetilde{\partial}_t)u(x,t) = \widetilde{\partial}_t^2 u - a^2 \widetilde{\partial}_t^4 u + b^2 \widetilde{\partial}_x^2 u - c^2 \widetilde{\partial}_x^2 u = X(x,t)/\rho$, where the displacement $u(x,t) \in C^4(\mathbb{R} \times \mathbb{R}_+)$, $X \in C^0(\mathbb{R} \times \mathbb{R}_+)$ represents the body force and a^2, b^2, c^2 are positive constants having the expressions $a^2 = \nu^2 r_0^2$, $b^2 = \frac{E\nu^2 r_0^2}{2(1+\nu)\rho}$, $c^2 = \frac{E}{\rho}$, where ν represents the Poisson ratio, ρ the density, E the modulus of elasticity, $r_0^2 = I/S$, I the inertial moment about the axis Ox and S the cross-section area.

Dirac Sequences and Electrodynamics

Using $D(\mathbb{R}^n)$ the Schwartz's space of indefinitely differentiable functions with compact support, and $D'(\mathbb{R}^n)$ the set of linear continuous functionals defined on $D(\mathbb{R}^n)$, then for $f_\varepsilon \in L_{loc}(\mathbb{R}^n)$ a Dirac sequence verifies in the sense of the convergence of $D'(\mathbb{R}^n)$ that $\lim_{\varepsilon \to 0} f_\varepsilon(x) = \delta(x)$, meaning that $\forall \varphi \in D(\mathbb{R}^n)$, $\lim_{\varepsilon \to 0}(f_\varepsilon(x), \varphi(x)) = (\delta(x), \varphi(x)) = \varphi(x)$. If $f_\varepsilon \in C^\infty(\mathbb{R}^n)$, then $\lim_{\varepsilon \to 0} D^\alpha f_\varepsilon(x) = D^\alpha \delta(x)$ with $D^\alpha f_\varepsilon(x) = \frac{\partial^{|\alpha|} f_\varepsilon(x_1, \ldots, x_n)}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \ldots \partial x_n^{\alpha_n}}$ representing the partial derivative of order $|\alpha| = \alpha_1 + \cdots + \alpha_n$ of the function f_ε. The study then addresses the characterization of Dirac sequences.

For $I_\varepsilon = (-\varepsilon, 0) \times \cdots \times (-\varepsilon, 0) \subset \mathbb{R}^n$, $\varepsilon > 0$, and $\chi_{I_\varepsilon}(x) = \begin{cases} 1, & x \in I_\varepsilon \\ 0, & x \notin I_\varepsilon. \end{cases}$, $x \in \mathbb{R}^n$ results that $\lim_{\varepsilon \to 0} \frac{1}{\varepsilon^n} \chi_{I_\varepsilon}(x) = \delta(x)$ and their applications in electrodynamics.

Dimensional analysis rules in the distribution space are developed, allowing singular electromagnetic sources to be treated consistently within generalized function calculus.

Fractional Calculus: Theory and Numerical Aspects

The second part of the work addresses fractional calculus, starting with the Caputo fractional derivative of order $\alpha \in (0,1)$:

$$D^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_0^x \frac{f^{(n)}(s)}{(x-s)^{\alpha+1-n}} ds \ , \quad n-1 < \alpha \leq n \ , \quad n \in \mathbb{N} \ ,$$

with $\alpha > 0$. This operator naturally incorporates memory effects, making it suitable for viscoelasticity, anomalous diffusion, and epidemiological models.
Fractional calculus represents the extension of derivatives and integrals to non-integer order derivatives and integrals, and has wide applicability in the world of science and engineering. It is the result of the famous correspondence between Leibniz and L'Hopital about the possible existence of the derivative of order 1/2. The most well-known fractional operator is the Riemann-Liouville integral operator is defined as

$$I^{\alpha} f(x) = \begin{cases} \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f(s)}{(x-s)^{1-\alpha}} ds = \frac{1}{\Gamma(\alpha)} t^{\alpha-1} * f(t) \ , & \alpha > 0 \ , \\ f(x) \ , & \alpha = 0 \ . \end{cases}$$

and $I^{\beta} \Theta^{\alpha}(x) \equiv \overline{\Theta}^{\alpha}(x,\beta)$, $f_{nm}^{\alpha}(x) = \begin{cases} F_m \left(N \ x^{\alpha} - n + 1\right) \ , & x \in \left[\left(\frac{n-1}{N}\right)^{1/\alpha}, \left(\frac{n}{N}\right)^{1/\alpha} \right) \ , \\ 0 \ , \quad \text{otherwise} \ , \end{cases}$

$$\overline{\Theta}^{\alpha}(x,\beta) = [I^{\beta} f_{10}^{\alpha}(x), \ldots, I^{\beta} f_{N0}^{\alpha}(x), I^{\beta} f_{11}^{\alpha}(x), \ldots, I^{\beta} f_{N1}^{\alpha}(x), \ldots, I^{\beta} f_{NM}^{\alpha}(x)]^T .$$

with $\alpha \in [0,1)$, $n = 1,2,\ldots,N$, and $m = 0,1,...,M$.
The accuracy of fractional numerical computations is investigated for several problem classes:

- Fredholm–Volterra integro-differential equations,

$$h_1(x \ , f(x) \ , D^{\alpha_1} f(x) \ , D^{\alpha_2} f(x) \ , \ldots \ , D^{\alpha_{\kappa}} f(x)) = \lambda h_2 \left(x \ , f(x) \ , \int_0^x p(x,s) G\left(s, f(s)\right) ds \right) \ ,$$

  subject to initial conditions $D^k f(0) = f_k \ , \quad k = 1,2,\ldots,\kappa \ ,$
  with $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_{\kappa} \geq 0$, with $\alpha_{\kappa} \in \mathbb{R}$, $0 \geq x \geq 1$ and $\lambda \in \mathbb{R}$.

- Riccati fractional differential equations,

$$a(x) D^{\beta} f(x) + b(x) f(x) + c(x) f^2(x) = g(x), \quad 0 \leq x \leq 1 \ ,$$

- fractional PDEs solved using hybrid function methods.

Numerical stability and error control are discussed, especially for long-time simulations where fractional orders are near integer boundaries.

Conclusions

Beyond mechanics and electrodynamics, fractional calculus is applied to: RLC circuits, structural health monitoring, Hamilton–Pfaff PDEs, epidemiology, and chaos theory. These examples demonstrate the wide applicability of fractional models in capturing phenomena that are inaccessible to integer-order models.

The combined use of distribution theory and fractional calculus creates a versatile analytical framework for modern engineering and physical sciences. Distribution theory rigorously addresses singular sources and discontinuities, while fractional calculus accounts for memory-dependent and nonlocal phenomena. The methodologies presented here extend classical modeling capabilities, offering robust solutions for problems once considered intractable.

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1. Partial Differential Equations

Computer Vision System for Animal Detection, Tracking and Behavior Analysis on Roads using Deep Learning and PDE Models

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This research, that is performed within a computer vision project in the road traffic monitoring domain, introduces a novel framework for multiple animal detection, classification and counting, which combines successfully some deep learning and mathematical models. The detection of animals in the traffic areas and monitoring their behavior represent important computer vision tasks that improve the road safety.
A deep learning-based animal detection technique is proposed first. We have created and annotated a voluminous animal image database containing 7 common classes of domestic and wild animals seen on or near roads, and split it into the training, validation and testing datasets. A convolutional neural network (CNN) - based animal detector is built by training, validating and testing a YOLO-v9 deep neural network on those 3 datasets. It locates successfully the animals in the frames of the traffic video sequence and classify them by their species.
The animal tracking (counting) process is performed by applying a tracking by detection (TBD) approach that treats both the appearance and motion of the detected animals. Their appearance is characterized by applying a multi-scale analysis-based high-level feature extraction on the detections using a nonlinear reaction-diffusion equation based scale-space and some deep networks. Their motion is treated by using an Intersection over Union (IoU) - based metric and the distances between the centroids. Some detection bounding box size related conditions are also used by the proposed TBD algorithm.
Then, one performs also a transfer learning-based video frame semantic segmentation, in order to identify the main components of the traffic scene: roadways, sidewalks and crosswalks. The animal detections and tracks that intersect these regions are determined next. Thus, some walking behaviors of these animals can be modeled. The proposed CV system could significantly improve the road safety by preventing the vehicle collisions and potential injuries to both people and animals. It can be applied successfully in the autonomous vehicle domain.

Keywords: *traffic video sequence, CNN-based animal detection, annotated animal database, tracking by detection algorithm, multi-scale high-level feature extraction, nonlinear diffusion model, IoU-based metric, video semantic segmentation.*

Acknowledgments. This research work was supported by a grant of the Romanian Academy, GAR-2023, Project Code 19.

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Determination of some solutions of the 3D Navier-Stokes equations

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The following systems of partial differential equations is examined:

$$\begin{cases} u_t + \dfrac{P_x}{\mu} + uu_x + vu_y + wu_z = \lambda\Delta u + F_x, \\ v_t + \dfrac{P_y}{\mu} + uv_x + vv_y + wv_z = \lambda\Delta v + F_y, \\ w_t + \dfrac{P_z}{\mu} + uw_x + vw_y + ww_z = \lambda\Delta u + F_z. \end{cases} \tag{1}$$

$$u_x + v_y + w_z = 0. \tag{2}$$

Here $P = P(t,x,y,z)$, $u = u(t,x,y,z)$, $v = v(t,x,y,z)$; $w = w(t,x,y,z)$; $F = F(t,x,y,z)$; $t > 0$, $u_x = \dfrac{\partial u}{\partial x}$; $\Delta u = u_{xx} + u_{yy} + u_{zz}$; $x, y, z \in R$.

Systems (1), (2) describes the process of the non-stationary flow of a liquid or gas in three-dimensional space. The system consists of (1), (2) represents the Navier-Stokes equations in the case of three-dimensional non-stationary motion of a viscous incompressible fluid or gas.

The P function represent the pressure of the liquid, and u, v, w functions represent the flow velocity component of the liquid or gas. The function F represents the external force and is potential in nature. The constants $\lambda > 0$ and $\mu > 0$ are a determined parameter of the studied liquid's (or gas) viscosity and density; $\lambda = \frac{c}{R_e}$, $c > 0$, where R_e is the Reynolds number.

Regarding the derivation of the equations of system (1), (2) and the meaning of the physical processes described by this system, consult the works [1]. A number of solutions to the equations in the stationary two- and three-dimensional case have been determined in the papers [2], [3] and [5]. Regarding the application of various methods for determining the solutions of nonlinear equations with partial derivatives, you can see the works [4].

The following theorem generates a series of solutions of equations (1), (2).

Theorem 1. *May it be $f = \varphi(t)a(x)b(y)c(z)$, where the functions $\varphi(t)$ is differentiable and $a(x)$, $b(y)$ and $c(z)$ are doubly differentiable and solutions of the following equations:*

$a'' = ra; b'' = sb; c'' = lc, r + s + l = 0$, *here r, s, l are constants.*

Then for the flow velocity components of the problem (1), (2) we have $u = f_x, v = f_y, w = f_z$.

The pressure P is $P = \mu\left[F - 0,5\left(u^2 + v^2 + w^2\right) - f_t + C\varphi'(t)\right]$.

We note that the solutions to problem (1), (2) determined in theorem 1 verify the following conditions: $u_y = v_x, u_z = w_x, w_y = v_z$; and $\Delta u = \Delta v = \Delta w = 0$.

Example 1. The components of velocity are: $u = 3\sin(5z)e^{3x+4y-t}$, $v = 4\sin(5z)e^{3x+4y-t}$, $w = 5\cos(5z)e^{3x+4y-t}$.

Another example of a solution to problem (1), (2) that verifies the mentioned conditions is:

Example 2. For the components of velocity, we have: $u = \alpha(t)xs^{-\frac{3}{2}}$, $v = \alpha(t)ys^{-\frac{3}{2}}$, $w = \alpha(t)zs^{-\frac{3}{2}}$, $s = x^2 + y^2 + z^2$. Here $\alpha(t)$ is an arbitrary differentiable function.

The pressure in this ca se is equal to $P = \mu\left[F - 0,5\left(u^2 + v^2 + w^2\right) + \alpha'(t)s^{-\frac{1}{2}}\right]$.

The following theorem gives us solutions in which the viscosity parameter λ participates explicitly.

Theorem 2. *May it be $u = f_1(s)$, $v = f_2(s)$, $w = f_3(s)$, $s = t + x + y + z$, where the functions $f_k, k = 1, 2, 3$ is differentiable up to and including the third order. Then for the flow velocity components, we have:*

$$u = \frac{-1}{3\,(C+1)}\left[(C_1 + C_3)\,s + C_2 + C_4 + \frac{3\lambda}{C+1}\,(C_1 + C_3) + \frac{C+1}{3\lambda}\,(C_5 + C_6)\,e^{\frac{(C+1)s}{3\lambda}}\right];$$

$$v = \frac{-1}{3\,(C+1)}\left[(2C_1 - C_3)\,s + 2C_2 + C_4 + \frac{3\lambda}{C+1}\,(2C_1 - C_3) + \frac{C+1}{3\lambda}\,(C_5 - C_6)\,e^{\frac{(C+1)s}{3\lambda}}\right];$$

$w = C - u - v$, *where $C \neq -1, C_0, C_1, C_2, C_3, C_4, C_5, C_6$ – constant.*

For pressure we get:

$$P = \mu\left[F - 3\lambda(u'(s) + v'(s)) + uv + 0,5\left(u^2 + v^2 - w^2\right)\right]. \tag{3}$$

Example 3. If $C = -2/3$, the components of velocity are:

$$u = -\left[(C_1 + C_3)s + C_2 + C_4 + 9\lambda(C_1 + C_3) + (9\lambda)^{-1}(C_5 + C_6)e^{\frac{s}{9\lambda}}\right];$$

$$v = -\left[(C_3 - 2C_1)s + 2C_2 - C_4 + 9\lambda(2C_1 - C_3) + (9\lambda)^{-1}(C_6 - 2C_5)e^{\frac{s}{9\lambda}}\right];$$

$$w = C - u - v, s = t + x + y + z.$$

The pressure P is determine from (3).

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On the convergence of an approximation scheme of fractional steps type, associated to a nonlinear second-order anisotropic reaction-diffusion SEIRD epidemic model

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In this paper, we investigate a reaction-diffusion problem with a cubic nonlinear reaction term and non-homogeneous Robin boundary conditions. We start our study by investigating the solvability of the problem in a proper class, proving the existence, uniqueness and the regularity of the solution. Our second goal is to develop an iterative splitting scheme to numerical treat the cubic nonlinearity. We propose a second-order fractional-steps method and results concerning the convergence of the numerical scheme are also established. We formulate a corresponding conceptual algorithm in order to approximate the solution of the nonlinear parabolic problem. Some computational experiments are made to illustrate the effectiveness of the proposed method, with the goal of saving the calculation costs. Comparisons are made with the results obtained by the iterative Newton method. The benefit of the splitting method aims at simplifying the process of numerical computations due to its decoupling feature.

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Three families of solutions of a nonlinear fourth order partial differential equation

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The paper is dedicated to the study of existence and continuous dependence to certain solutions of non-dissipative double-dispersive microstrain wave model. The model we are interested in is governed by a fourth order nonlinear partial differential equation, that is

$$\begin{cases} \partial_{t,t}u(t,x) + \gamma_1\partial_{x,x}u(t,x) + \gamma_2\partial_{x,x}u^2(t,x) \\ \quad +\gamma_3\partial_{x,x,x,x}u(t,x) + \gamma_4\partial_{x,x,t,t}u(t,x) = 0. \end{cases}$$

Many interesting results have been published using different methods to approach the solutions: soliton solutions, kink solutions, antikink solutions, and bell shaped solutions. In the present paper we use *Mathematica* to find a general soliton solution, a large family of soliton solution, as well as kink and antikink solutions. The families of solutions are obtained by the Tanh ansatz method.

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2. ODEs; Dynamical Systems

Dynamics and Bifurcations of Logistic Models Applied to Water Electrolysis for Hydrogen Production

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The paper presents a nonlinear mathematical model for the dynamics of an electrolyzer, incorporating a logistic coupling for the ionic concentration at the electrode and nonlinear feedback between current and temperature:

$$\begin{cases} \frac{dC}{dt} = rC\left(1 - \frac{C}{K}\right) - \alpha I \\ \\ \frac{dI}{dt} = \beta C^n V - \delta I - \eta(T - T_0)I \\ \\ \frac{dT}{dt} = \mu I^2 - \lambda(T - T_{amb}) \end{cases}$$

The proposed model captures essential features of real electrochemical systems, including saturation effects and thermal feedback, which are often neglected in classical linear approaches. Through analytical and numerical analysis, we identify the equilibrium points and study their stability by evaluating the eigenvalues of the associated Jacobian matrix. The results reveal the existence of both saddle-node and Hopf bifurcations as the applied voltage is varied, delineating the transition between stable, oscillatory, and unstable operating regimes. Critical thresholds for these bifurcations are highlighted, providing valuable insights for the design and control of electrolyzers. The study shows the necessity of considering nonlinearities and thermal effects in mathematical models to accurately predict and optimize the behavior of advanced electrochemical systems.

Quadratic polynomial differential systems possessing at least one invariant ellipse and at least one parabola

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This work concerns the family QS of quadratic systems:

$$\dot{x} = p_0 + p_1(x, y) + p_2(x, y) \equiv P(x, y),\ \dot{y} = q_0 + q_1(x, y) + q_2(x, y) \equiv Q(x, y),$$

where p_i and q_i are homogeneous polynomials (i = 0, 1, 2) of degree i in x, y.
We are interested in the family of non-degenerate systems belonging to QS possessing affine algebraic invariant curves.
In the article [1] (respectively in [2]), the classification in terms of affine invariant polynomials of

the subfamily of systems *QSE* (respectively *QSP*) possessing at least one invariant ellipse (respectively parabola) was performed.
We are interesting in the classification of subfamily *QSEP* of quadratic systems possessing simultaneously an invariant ellipse and a parabola. More exactly we determine all the configurations of such invariant curves which can possess a system in *QSEP*.
Following [3] we call *configuration of invariant algebraic curves* of a planar polynomial system, the set of (complex) invariant algebraic curves (which may have real coefficients) of the system, each endowed with its own multiplicity and together with all the real singular points of this system located on these invariant curves, each one endowed with its own multiplicity.
Our results are the following ones:

1. We prove the existence of exactly seven distinct configurations of invariant curves of degree ≤ 2 for systems in *QSEP*, possessing a finite number of singularities (finite or infinite).

2 For systems belonging to *QSEP* possessing a line of singularities at infinity we determine the existence of exactly three distinct configurations which beside the invariant parabola contain an one-parameter family of invariant ellipses.

3 Necessary and sufficient affine invariant conditions for the realization of each one the mentioned configurations are constructed, in terms of polynomial invariants.

Acknowledgement. *This work is partially supported by the Program SADGET 011303 and partially by the grant 25.80012.5007.83SE, Moldova State University.*

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On a sequential fractional differential inclusion

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We consider the following problem

$$\mu D_C^q x(t) + \xi D_C^{q-1} x(t) \in F(t, x(t)) \quad a.e.\ ([a,b]), \tag{1}$$

$$x(a) = \sigma, \quad x'(a) = x(b) = 0, \quad x'(b) = \sum_{i=1}^{m} \alpha_i x(\eta_i) + \int_a^b x(s)ds, \tag{2}$$

where $q \in (3,4]$, $a < \eta_1 < \eta_2 < ... < \eta_m < b$, $\sigma, \mu, \xi, \alpha_i \in \mathbf{R}$, $i = \overline{1,m}$, $\mu, \xi \neq 0$, D_C^q is the Caputo fractional derivative of order q and $F : [a,b] \times \mathbf{R} \to \mathcal{P}(\mathbf{R})$ is a set-valued map.

Our aim is to extend study in [1] to the more general case when the right-hand side of (1) is set-valued. We have two objectives. On one hand, we provide two existence results for the problem above. The first one is done by using the Covitz and Nadler set-valued contraction principle ([2]) and the second one relies on Filippov's ideas ([3]). On the other hand, by using a result ([4]) concerning the arcwise connectedness of the fixed point set of a class of set-valued contractions, we prove the arcwise connectedness of the solution set of problem (1)-(2).

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Opial's Theorem for Scalar Remotely Almost Periodic Differential Equations

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Denote by $\mathbb{R} := (-\infty, +\infty)$ and $C(\mathbb{R},\mathbb{R})$ (respectively, $C(\mathbb{R}\times\mathbb{R},\mathbb{R})$ the space of all continuous functions $\varphi : \mathbb{R} \to \mathbb{R}$ (respectively, $f : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$) equipped with the compact-open topology. Let $(C(\mathbb{R},\mathbb{R}),\mathbb{R},\sigma)$ (respectively, $(C(\mathbb{R}\times\mathbb{R},\mathbb{R}),\mathbb{R},\sigma)$) be the shift dynamical system on the space $C(\mathbb{R},\mathbb{R})$ (respectively, on the space $C(\mathbb{R}\times\mathbb{R},\mathbb{R})$).

Recall that a function $\varphi \in C(\mathbb{R},\mathbb{R})$ is said to be

1. almost periodic [1] if for any $\varepsilon > 0$ there exists a positive number $l = l(\varepsilon)$ such that on every segment $[a, a+l]$ there exists at least one number τ for which $|\varphi(t+\tau) - \varphi(t)| < \varepsilon$ for any $t \in \mathbb{R}$;

2. asymptotically almost periodic [1] if there are $p, r \in C(\mathbb{R},\mathbb{R})$ such that $\varphi = p + r$, p is almost periodic and $|r(t)| \to 0$ as $t \to +\infty$;

3. remotely almost periodic [2,3] if for every $\varepsilon > 0$ there exists a positive number $l = l(\varepsilon)$ such that on every segment $[a, a+l]$ there exists at least one number τ and $L(\varepsilon,\tau) > 0$ so that $\rho(\varphi(t+\tau),\varphi(t)) < \varepsilon$ for any $t \geq L(\varepsilon,\tau)$.

Remark 1. 1. Every almost periodic (respectively, asymptotically almost periodic) function is asymptotically almost periodic (respectively, remotely almost periodic). The converse statement is false.

2. The notion of remotely almost periodicity on the real axis $\mathbb{R}$ for the scalar functions was introduced and studied by Sarason D. (1984). Remotely almost periodic functions on the semi-axis $\mathbb{R}_+$ with the values in the Banach space were introduced and studied by Ruess W. M. and Summers W. H. (1986). Remotely almost periodic functions on the real axis with the values in the Banach spaces were introduced and studied by Baskakov A. G. (2013). He calls theses functions "almost periodic at infinity".

A function $\varphi \in C(\mathbb{R}, \mathbb{R})$ is said to be remotely τ-periodic if $\lim\limits_{t \to +\infty} |\varphi(t+\tau) - \varphi(t)| = 0$.

Example. The functions $\varphi \in C(\mathbb{R}, \mathbb{R})$ defined by $\varphi(t) := \cos t + \sin \ln(1 + |t|)$ for any $t \in \mathbb{R}$ is remotely τ-periodic [3].

Remark 2. Every remotely τ-periodic function is remotely almost periodic. The converse statement is false.

A function $f \in C(\mathbb{R} \times \mathbb{R}, \mathbb{R})$ is said to be regular (respectively, positively regular) if for every $g \in H(f)$ (respectively, $g \in H^+(f)$) and $v \in \mathbb{R}$ there exists a unique solution $\varphi(t, v, g)$ of the equation $y' = g(t, v)$ passing through the point $v \in \mathbb{R}$ at the initial moment $t = 0$ and defined on $\mathbb{R}_+ := [0, +\infty)$, where $H(f)$ (respectively, $H^+(f)$) is the closure of the set of all translations $\{f^h |\ h \in \mathbb{R},\ f^h(t, x) := f(t + h, x)$ for any $(t, x) \in \mathbb{R} \times \mathbb{R}\}$ (respectively, $\{f^h |\ h \geq 0,\ f^h(t, x) := f(t + h, x)$ for any $(t, x) \in \mathbb{R} \times \mathbb{R}\}$) in the space $C(\mathbb{R} \times \mathbb{R}, \mathbb{R})$.

It is well known the following result.

Theorem 1 (Opial [4])**.** *Assume that the following conditions are fulfilled:*

1. *the function $f \in C(\mathbb{R} \times \mathbb{R}, \mathbb{R})$ is almost periodic in time t uniformly with respect spacial variable x on every compact subset from $\mathbb{R}$;*
2. *the function f is monotone with respect to x, i.e., $x_1 \leq x_2$ implies $f(t, x_1) \leq f(t, x_2)$ for all $t \in \mathbb{R}$ and $x_1, x_2 \in \mathbb{R}$;*
3. *the equation*
$$x'(t) = f(t, x) \tag{1}$$
admits a bounded on $\mathbb{R}_+$ solution $\varphi(t, u_0, f)$.

Then there exist at least one almost periodic solution $\varphi(t, p, f)$.

Question. Is the Opial's theorem true for asymptotically almost periodic differential equations?

Unfortunately, the answer to this question (in general) is negative.

Our main result in the following theorem is contained.

Theorem 2. *Suppose that the following conditions hold:*

1. *the function $f \in C(\mathbb{R} \times \mathbb{R}, \mathbb{R})$ is asymptotically almost periodic in time, i.e., there are functions $p, r \in C(\mathbb{R} \times \mathbb{R}, \mathbb{R})$ such that*
   (a) *$f(t, x) = p(t, x) + r(t, x)$ for any $(t, x) \in \mathbb{R} \times \mathbb{R}$;*
   (b) *the function p is almost periodic in time t and $\lim\limits_{t \to +\infty} |r(t, x)| = 0$ uniformly with respect to x on every compact subset from $\mathbb{R}$;*
2. *f is positively regular;*
3. *the function f is monotone with respect to x uniformly in $t \in \mathbb{R}$;*
4. *the equation (1) admits a bounded on $\mathbb{R}_+$ solution $\varphi(t, u_0, f)$.*

Then the solution $\varphi(t, u_0, f)$ is remotely almost periodic.

Remark 3. Theorem 2 remains true if the asymptotic almost periodicity (in time) we replace by remote almost periodicity of f under the condition that the ω-limits set ω_f is minimal.

Open problem. The question is whether Theorem 2 remains true for remotely almost periodic differential equations in the general case, when the set ω_f is not minimal, it remains open.

Acknowledgement. *This research was supported by the State Programs of the Republic of Moldova "Monotone Nonautonomous Dynamical Systems (24.80012.5007.20SE)", "Remotely Almost Periodic Solutions of Differential Equations (25.80012.5007.77SE)" and partially was supported by the Institutional Research Program 011303 "SATGED", Moldova State University.*

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The problem of the center for a cubic differential system with one invariant straight line and one invariant cubic

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We consider the cubic system of differential equations

$$\dot{x} = y + P_2(x, y) + P_3(x, y),\ \dot{y} = -x + Q_2(x, y) + Q_3(x, y), \tag{1}$$

where $P_j(x, y)$, $Q_j(x, y) \in \mathbb{R}[x, y]$, $j \in \{1, 2\}$ are homogeneous polynomials of degree j. The origin $O(0, 0)$ is a singular point for (1) with purely imaginary eigenvalues, i.e. a focus or a center. The problem of distinguishing between a center and a focus (the problem of the center) is open for general cubic systems.

The problem of the center was solved for system (1) with: four invariant straight lines in [1], [2]; three invariant straight lines in [1]; two parallel invariant straight lines in [3]; two invariant straight lines and one invariant conic in [1]; two invariant straight lines and one invariant cubic in [4]. By using the method of Darboux integrability, the center conditions were found for system (1) with one algebraic solution in [5] and with two algebraic solutions in [6].

We study the problem of the center for a cubic differential system (1) assuming that the system has one invariant straight line and one invariant cubic. In this talk we consider the following tasks:

(i) Find the subclass of cubic differential systems (1) which has one invariant straight line $1 + Ax + By = 0$ and one invariant cubic $x^2 + y^2 + a_{30}x^3 + a_{21}x^2y + a_{12}xy^2 + a_{03}y^3 = 0$ intersecting at a singular point (x_0, y_0);

(ii) For this subclass determine the center conditions.

It is proved that the cubic differential system (1) with one invariant straight line and one invariant cubic intersecting at a singular point (x_0, y_0) has a center at the origin $O(0, 0)$ if and only if the first five Lyapunov quantities vanish.

Acknowledgement. *This work was supported by the National Agency for Research and Development of the Republic of Moldova under project number* 25.80012.5007.76SE Qualitative and algebraic investigation of differential models.

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On a Model of Fake News Spreading Through Social Media

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Fake news have become a serious issue in today's modern era, particularly in view of increased activity in social and online platforms. False information can go viral almost instantaneously through social networks that immediately affect society and people's minds.

The form of fake news it develops within, whether fabricated intentionally or not, impacts public perspectives through manipulation of emotion and cognition.

We propose and analyze a mathematical model describing how fake news can spread through an online social media (OSM) platform. The model presents a generalized context of several fake news items with independent delay and skepticism parameters. Our results provide some conditions under which fake news either die out or become persistent, and they show the influence of delays, skepticism levels, and incidence rates on the dynamics of information spread.

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Off-label use of epidemic modelling: assessing academic influence among your peers

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We examine the influence of social-cognitive factors such as self-efficacy, locus of control and negative peer pressure on the academic performance and dropout intentions of undergraduate students. To this purpose, an epidemiological modelling framework is employed, of concern being a 4-dimensional model which bears resemblance to a SEI model with two stages of infectivity.

Sufficient conditions for the existence and stability of the equilibria are determined in terms of threshold parameters defined adhoc, similar in scope, definition and interpretation to the basic reproduction number of an epidemic model.
After performing further numerical simulations to support and explore the relevance of our theoretical findings, we observe that the model exhibits rich dynamical behavior, such as multiple positive equilibria, backward bifurcations (which means that resit and dropout can become mainstays even if the resit reproduction number is less than one), transcritical bifurcations and Hopf bifurcations leading to oscillatory solutions.

Malaria dynamics with bimodality of incubation period in hosts in a seasonal environment

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To describe the bimodal distribution of the incubation time of *P. vivax* malaria in Korea, corresponding to empirical observations, we present a periodic compartmental model of delay differential equations for malaria transmission dynamics with two distinct exposed classes in the human population and including time-dependent parameters for mosquito birth and death rates as well as biting rates. The short-term incubation period is modeled by an exponential distribution, while the long-term incubation is assumed to be of fixed length. We identify the basic reproduction number as the spectral radius of a linear operator and demonstrate that it serves as a threshold parameter for the global dynamics of the model. We apply the model to data from South Korea. Our numerical simulations support the analytical results.

On the homogenization of polynomial dynamical systems in the plane. The case of systems arising from excitable media

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The problem of stabilization of dynamical systems is very challenging, as part of the control systems' field. The theory of positive polynomials in control has the seeds in the 1980's, based on the work of Naum Zuselevich. They can be used to solve a variety of problems in robust control, non-linear control and also in non-convex optimization. The present paper approaches the problem of finding a stabilizing feedback for homogeneous polynomial systems, in the plane. It is known that the polynomial systems in the plane have a lot of special properties which can be easier approached thanks to the dimension two. The case of systems arising from excitable media, namely the kinematics of a mixing flow, is taken into account. The results will be used to deduce properties for further detailed analysis.

On the Number of Center-Affine Invariant Elements for the Differential System $s^3(1,3)$

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We will examine the differential system $s^3(1,3)$ in the tensorial form

$$\frac{dx^j}{dt} = a^j_\alpha x^\alpha + a^j_{\alpha\beta\gamma} x^\alpha x^\beta x^\gamma \ (j,\alpha,\beta,\gamma = \overline{1,3}), \tag{1}$$

where the coefficient tensor $a^j_{\alpha\beta\gamma}$ is symmetrical in lower indices in which the complete convolution holds. In [1] was determined the 9-dimensional Lie algebra of operators of the representation of the centro-affine group of transformations $GL(3,\mathbb{R})$ in the space of variables and coefficients of the system (1).

Based on the ideas from the papers [2–7], some ternary differential systems were examined in the works [7–9] under different aspects. However, for ternary differential systems, the problem of constructing the functional bases of invariants, comitants, contravariants and mixed comitants (see, for example [2–8]), is not completely solved.

By means of the Lie algebras of the differential operators, admitted by the differential systems $s^3(1)$, $s^3(3)$ and $s^3(1,3)$, it was proved

Theorem 1. *The number of the invariant center-affine polynomials of the differential systems $s^3(1)$, $s^3(3)$ and $s^3(1,3)$ that form the functional bases of invariants $\mu(I)$, covariants $\mu(K)$, contravariant $\mu(R)$ and of mixed comitants $\mu(S)$, respectively, is given in the following table*

	$s^3(1)$	$s^3(3)$	$s^3(1,3)$
$\mu(I)$	*3*	*22*	*31*
$\mu(K)$	*4*	*25*	*34*
$\mu(R)$	*4*	*25*	*34*
$\mu(S)$	*7*	*28*	*37*

Acknowledgement. *This work was supported by the Institutional Research Program 011303 "SATGED" Moldova State University.*

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On the Jacobi Stability of a 3D Chaotic System

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In this work, a three dimensional quadratic polynomial autonomous smooth chaotic system is considered. By reformulating the first-order differential system as a system with two second-order differential equations, we will investigate the nonlinear dynamics of the system from the Jacobi stability perspective through the Kosambi-Cartan-Chern (KCC) geometric theory. The intrinsic geometric properties of the systems will be studied by determining the associated geometric objects: the zero-connection curvature tensor, the nonlinear connection, the Berwald connection, and the five KCC invariants (the external force - the first invariant; the deviation curvature tensor - the second invariant; the torsion tensor - the third invariant; the Riemann–Christoffel curvature tensor - the fourth invariant; the Douglas tensor - the fifth invariant). In order to obtain necessary and sufficient conditions for the Jacobi stability near each equilibrium point, the deviation curvature tensor will be determined at each equilibrium point. Furthermore, in order to deepen our knowledge about the dynamics of this chaotic dynamical system, we will compare the Jacobi stability with the classical Lyapunov (linear) stability.

Keywords: chaotic system, Jacobi stability, KCC-theory
2020 MSC : 34D20, 37C20, 37C75, 53E10

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Lie algebra of the differential operators of $GL(3,R)$ group, admitted by the differential system $s^3(1,4)$ and the functional bases of the invariants of this system

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We consider the differential system $s^3(1,4)$ of the form

$$\frac{dx^j}{dt} = a^j_\alpha x^\alpha + a^j_{\alpha\beta\gamma\delta} x^\alpha x^\beta x^\gamma x^\delta \quad (j,\alpha,\beta,\gamma,\delta = \overline{1,3}), \tag{1}$$

where $a^j_{\alpha\beta\gamma\delta}$ is a symmetric tensor in lower indices in which the total convolution is done and the center-affine group $GL(3,R)$ of transformations of the contravariant vector $x = (x^1, x^2, x^3)$ [1], whose coordinates represent the phase variables of the system (1). We also consider the covariant vector $u = (u_1, u_2, u_3)$ [1], whose coordinates change according to the inverse matrix of the center-affine transformation of the vector x.
It is easy to see that the system (1) contains, as particular cases, the differential systems $s^3(1)$ and $s^3(4)$ with homogeneities of degree 1 and 4, respectively.
Some particular cases on integrability and stability of unperturbed motion were studied for system (1) in the works [8-11], without using the functional bases of invariant polynomials, which could

fully characterize these properties.
In this paper, using the methods developed in the works [1-7], it was constructed the Lie algebra of representation operators of the center-affine group $GL(3,R)$ in the space $E^{60}(a,x,u)$ of the coefficients a of system (1) and of the coordinates of the vectors x and u.
By means of this algebra, it was proved:

Theorem 1. *The number of the center-affine invariant polynomials of the systems $s^3(1)$, $s^3(4)$, $s^3(1,4)$ that form the functional bases of invariants $\mu(I)$, covariants $\mu(K)$, contravariant $\mu(R)$ and of mixed comitants $\mu(S)$ (see, for example [1,6]), is given in the following table*

	$\mu(I)$	$\mu(K)$	$\mu(R)$	$\mu(S)$
$s^3(1)$	*3*	*4*	*4*	*7*
$s^3(4)$	*37*	*40*	*40*	*43*
$s^3(1,4)$	*46*	*49*	*49*	*52*

Some elements of these bases were constructed.

Acknowledgement. *This work was supported by the Institutional Research Program 011303 "SATGED" Moldova State University.*

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The Lie algebra of the ternary differential system of Lyapunov-Darboux type $s^3(1,6)$

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We consider the ternary differential system $s^3(1,6)$ of the form

$$\frac{dx^j}{dt} = a^j_\alpha x^\alpha + a^j_{\alpha\beta\gamma\delta\mu\nu} x^\alpha x^\beta x^\gamma x^\delta x^\mu x^\nu \ \ (j,\alpha,\beta,\gamma,\delta,\mu,\nu = \overline{1,3}), \tag{1}$$

where $a^j_{\alpha\beta\gamma\delta\mu\nu}$ is a symmetric tensors in the lower indices, by which a total convolution is carried out [1], [2].

Lemma 1. *Let the invariant conditions [3]*

$$K_{3,3} \not\equiv 0, \ \ L_{1,3}L_{2,3} = L_{3,3}$$

be satisfied. Then by a center-affine transformation, the system (1) can be brought to the canonical form of Lyapunov-Darboux type [3], [4]:

$$\begin{aligned} \frac{dx^1}{dt} &= -\lambda x^2 + 6x^1 R(x^1,x^2,x^3) \equiv P^1, \\ \frac{dx^2}{dt} &= \lambda x^1 + 6x^2 R(x^1,x^2,x^3) \equiv P^2, \\ \frac{dx^3}{dt} &= x^2 - L_{1,3}x^3 + 6x^3 R(x^1,x^2,x^3) \equiv P^3, \end{aligned} \tag{2}$$

where $K_{3,3} = a^\alpha_\mu a^\beta_\delta a^\gamma_\alpha x^\delta x^\mu x^\nu \varepsilon_{\beta\gamma\nu}$, $L_{1,3} = -I_{1,3}$, $\lambda^2 = L_{2,3} = \frac{1}{2}(I^2_{1,3} - I_{2,3}) > 0$, $L_{3,3} = \frac{1}{6}(-I^3_{1,3} + 3I_{1,3}I_{2,3} - 2I_{3,3})$, $I_{1,3} = a^\alpha_\alpha$, $I_{2,3} = a^\alpha_\beta a^\beta_\alpha$, $I_{3,3} = a^\alpha_\gamma a^\beta_\alpha a^\gamma_\beta$, *and*

$$\begin{gathered} R(x^1,x^2,x^3) = a_1(x^1)^5 + 5a_2(x^1)^4x^2 + 5a_3(x^1)^4x^3 + 10a_4(x^1)^3(x^2)^2 + \\ +20a_5(x^1)^3x^2x^3 + 10a_6(x^1)^3(x^3)^2 + 10a_7(x^1)^2(x^2)^3 + 30a_8(x^1)^2(x^2)^2z + \\ +30a_9(x^1)^2x^2(x^3)^2 + 10a_{10}(x^1)^2(x^3)^3 + 5a_{11}x^1(x^2)^4 + 20a_{12}x^1(x^2)^3x^3 + \\ +30a_{13}x^1(x^2)^2(x^3)^2 + 20a_{14}x^1x^2(x^3)^3 + 5a_{15}x^1(x^3)^4 + a_{16}(x^2)^5 + 5a_{17}(x^2)^4x^3 + \\ +10a_{18}(x^2)^3(x^3)^2 + 10a_{19}(x^2)^2(x^3)^3 + 5a_{20}x^2(x^3)^4 + a_{21}(x^3)^5. \end{gathered}$$

Solving the determining equations [4]

$$\begin{aligned} (\xi^1)_{x^1}P^1 + (\xi^1)_{x^2}P^2 + (\xi^1)_{x^3}P^3 &= \xi^1 P^1_{x^1} + \xi^2 P^1_{x^2} + \xi^3 P^1_{x^3}, \\ (\xi^2)_{x^1}P^1 + (\xi^2)_{x^2}P^2 + (\xi^2)_{x^3}P^3 &= \xi^1 P^2_{x^1} + \xi^2 P^2_{x^2} + \xi^3 P^2_{x^3}, \\ (\xi^3)_{x^1}P^1 + (\xi^3)_{x^2}P^2 + (\xi^3)_{x^3}P^3 &= \xi^1 P^3_{x^1} + \xi^2 P^3_{x^2} + \xi^3 P^3_{x^3}, \end{aligned}$$

where P^j $(j = \overline{1,3})$ are from (2), and $\xi^i = A^i_\alpha x^\alpha + A^i_{\alpha\beta\gamma\delta\mu\nu} x^\alpha x^\beta x^\gamma x^\delta x^\mu x^\nu$ $(i, \alpha, \beta, \gamma, \delta, \mu, \nu = \overline{1,3})$ are the coordinates of the operators $X = \xi^i \frac{\partial}{\partial x^i}$ $(i = \overline{1,3})$, we obtain a Lie algebra of three independent linear operators X_1, X_2 and X_3. It was proved

Theorem 1. *The operators X_1, X_2, X_3 form a commutative three-dimensional Lie algebra L_3 admitted by the Lyapunov-Darboux type differential system (2).*

Acknowledgement. *This work was supported by the National Agency for Research and Development of the Republic of Moldova under project number* 25.80012.5007.76SE Qualitative and algebraic investigation of differential models.

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Multiplicity of the line at infinity in degree-six planar polynomial systems

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We study 6th degree planar polynomial differential systems

$$\begin{cases} \dot{x} = \sum\limits_{n=0}^{6} \sum\limits_{i=0}^{n} a_{i,n-i}\, x^i y^{n-i}, \\ \dot{y} = \sum\limits_{n=0}^{6} \sum\limits_{i=0}^{n} b_{i,n-i}\, x^i y^{n-i}, \end{cases} \tag{1}$$

with real coefficients $a_{i,j}, b_{i,j}$.
Our focus is on systems whose invariant straight line at infinity attains a high multiplicity in the framework of the Poincaré compactification. We determine algebraic conditions ensuring a certain minimal multiplicity and provide explicit examples of systems satisfying them. Some cases leading to extremely cumbersome computations are omitted, although we outline possible approaches to address them in future work, aiming to determine the maximal multiplicity. The results extend previous classifications for quadratic, cubic, quartic, and quintic systems [1], and reveal new algebraic configurations and dynamical behaviours specific to the degree-six case.

Acknowledgement. *This work was supported by the National Agency for Research and Development of the Republic of Moldova, project number* 25.80012.5007.76SE Qualitative and algebraic investigation of differential models

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Existence of solution for q-Fractional Differential Equations with Boundary Conditions in Banach Spaces

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In this work, we discuss the solutions to the boundary value problem for fractional q-difference equations with nonlinear integral conditions:

$$ {}^{c}D_{q,\gamma}\nu(\iota) - \lambda h(t,\nu(\iota)) = \mathcal{J}(\iota,\nu(\iota),{}^{c}D_{q,\gamma}\nu(\iota)),\ \iota\in[0,\ell],1<\alpha\leq 2, \tag{1.1} $$

$$ \nu(0)-\nu'(0)=c_1\int_0^{\ell}\mathcal{G}_1(\tau,\nu(\tau))\,d_q\tau, \tag{1.2} $$

$$ \nu(\ell)-\nu'(\ell)=c_2\int_0^{\ell}\mathcal{G}_2(\tau,\nu(\tau))\,d_q\tau, \tag{1.3} $$

By applying the fixed point theorems of Banach and Krasnoselskii, we establish the existence of solutions for the above problem (1.1) – (1.3). Some illustrative examples are giveen.

Introduction

Over the past years. fractional differential calculus attracted the interest of several academics, due to it's important for mathematical modeling. Its applications can be observed in domains such as physics, engineering, biology, and finance, where standard calculus often fails, see $[9-11,15,17]$ and the references therein.

One of the fractional differential equations that has attracted a lot of attention from researchers is quantum difference calculus; see $[1,3,4,7,12-14,18]$ and its references.

In this research, we investigate the existence of solutions to the boundary value problem (BVP) for fractional q-difference equations with nonlinear integral conditions.

$$ {}^{c}D_{q,\gamma}\nu(\iota) - \lambda h(t,\nu(\iota)) = \mathcal{J}(\iota,\nu(\iota),{}^{c}D_{q,\gamma}\nu(\iota)),\ \iota\in[0,\ell],1<\alpha\leq 2, \tag{1.1} $$

$$ \nu(0)-\nu'(0)=c_1\int_0^{\ell}\mathcal{G}_1(\tau,\nu(\tau))\,d_q\tau, \tag{1.2} $$

$$ \nu(\ell)-\nu'(\ell)=c_2\int_0^{\ell}\mathcal{G}_2(\tau,\nu(\tau))\,d_q\tau, \tag{1.3} $$

where $q\in(0,1)$ and λ,c_1 and $c_2\in\mathbb{R}$, $\mathcal{G}_1,\mathcal{G}_2$, $h:[0,\ell]\times\mathcal{E}\to\mathcal{E}$, and $f:[0,\ell]\times\mathcal{E}\times\mathcal{E}\to\mathcal{E}$ are continuous functions, wich $\mathcal{E}$ is a reflexive Bnach space. ${}^{c}D_{q,\gamma}$ represents Caputo fractional $q-$difference derivative of order γ.

Keywords: Caputo fractional q-difference inclusion; Fixed point theorems; Integral boundary conditions.

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A multiple degenerate Hopf-Hopf bifurcation. Application to a finance model

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The nondegenerate Hopf-Hopf bifurcation and some simple degenerate cases are emphasized. A new double degenerate Hopf-Hopf bifurcation is presented. Its normal form and bifurcation diagram are obtained. An application of this bifurcation to an economic system is done.

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Stabilization of the nonlinear Korteweg-de Vries equation with delay

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The Korteweg-de Vries equation is a nonlinear dispersive partial differential equation which models the propagation of long waves in water of shallow depth. In this paper, we consider the nonlinear Korteweg-de Vries equation in the case where we have a boundary feedback with a time delay and an internal feedback without delay. We impose some assumptions on the feedbacks. We first prove the existence and uniqueness of the solution of the system using the semigroups theory and we prove the exponential stability result, using a Lyapunov functional approach for any length of the spatial domain

Keywords: Stabilization, Nonlinear KdV equation, Time-delay, Lyapunov functional.

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Quartic differential systems with a center-focus critical point, an affine invariant straight line, and the line at infinity of maximal multiplicity

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We consider the real quartic system of differential equations having a singular point M with purely imaginary eigenvalues and an affine real invariant straight line l_1, where $M \notin l_1$. Using affine transformations and time rescaling, the system can be written in the form:

$$\begin{cases} \dot{x} = (1-x)(y + a_{20}x^2 + (1+a_{11})xy + a_{02}y^2 + (a_{20}+a_{30})x^3 + \\ +(1+a_{11}+a_{21})x^2y + (a_{02}+a_{12})xy^2 + a_{03}y^3) \equiv p(x,y), \\ \dot{y} = -(x + b_{20}x^2 + b_{11}xy + b_{02}y^2 + b_{30}x^3 + b_{21}x^2y + b_{12}xy^2 + \\ +b_{03}y^3 + b_{40}x^4 + b_{31}x^3y + b_{22}x^2y^2 + b_{13}xy^3 + b_{04}y^4) \equiv q(x,y). \end{cases} \tag{1}$$

For system (1), the invariant straight line l_1 is parallel to the ordinate axis and the singular point M, which is either a center or a focus, is placed at the origin.

In this paper, we determine the maximal multiplicity of the line at infinity for quartic system (1), assuming that the system (1) has at most five distinct singular points at infinity and that its right-hand sides have no common divisors of degree greater than zero.

Theorem 1. *For the quartic system* (1)*, the maximal multiplicity of the line at infinity is six. The line at infinity has maximal multiplicity for quartic system* (1) *if and only if one of the following two sets of conditions holds:*

1) $a_{03} = a_{12} = a_{21} = a_{30} = 0,\ a_{02} = 0,\ a_{11} = -1,\ a_{20} = 0,\ b_{04} = b_{13} = b_{22} = b_{40} = 0,\ b_{31} \neq 0,\ b_{30} = b_{12} = b_{03} = b_{20} = 0,\ b_{02} = -2;$

2) $a_{03} = a_{12} = a_{21} = 0,\ a_{30} = -a_{20},\ \ a_{02} = 0,\ a_{11} = -1,\ a_{20} \neq 0,\ b_{04} = b_{13} = b_{22} = b_{31} = 0,\ b_{40} = -2a_{20}^2,\ b_{03} = b_{12} = 0,\ b_{21} = -3a_{20},\ b_{30} = 0,\ b_{02} = -\dfrac{1}{2},\ b_{11} = 2a_{20}.$

Acknowledgement. *This work was supported by the National Agency for Research and Development of the Republic of Moldova under project number* "25.80012.5007.76SE Qualitative and algebraic investigation of differential models.

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3. Mathematical Modeling

Modeling and Optimization of MR Fluid-Based Oleo-Pneumatic Dampers for Aircraft Landing Gear

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This study presents the modelling of a pneumatic oleo damper integrated with magnetorheological (MR) fluid for application in aircraft landing gear systems. Initially, a damping element reported in the literature is analysed, and landing scenarios of the aircraft under various operating conditions are simulated. To mitigate the critical impacts during landing, an objective function utilizing Integral Square Error (ISE) is employed to optimize the suspension parameters. Given the presence of numerous parameters and to reduce computational complexity, the Particle Swarm Optimization (PSO) algorithm, recognized as one of the most efficient optimization techniques, is employed. The optimized parameters derived from this process are compared with the original parameters through simulations, and the results are presented in the form of graphs and tables. The study clearly demonstrates that, as a result of the optimization process, suspension performance, passenger comfort, and road-holding capacity have been significantly improved. Consequently, the safety of aircraft landings has been enhanced, and the service life of aircraft components has been prolonged.

Asymptotic and Numerical Approaches to Capillary Flow through Narrow Domains

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The first part of the paper is dedicated to the analytical description of capillary flow, as well as the Jurin, Washburn and Lucas-Washburn laws with inertia and the basic principles of fluid flow through micro-nozzle-type domains. The paper continues with the determination of approximate solutions to the numerical modelling of the given problem, by describing the initial and boundary conditions, the choice of physical parameters and the convergence analysis. In the last part, the convergence and accuracy of the results in both approaches are discussed. Conclusions and references are added.

Keywords: Capillary flow, asymptotical development, applied mathematics

Closed-loop model identification of the identical elements with inertia based on the underdamped step response

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In this paper is proposed an algorithm for experimental model identification of the identical elements with inertia second order based on the underdamped step response of the closed-loop system with P controller [1-2]. The system is proposed to be approximated with the following transfer function:

$$H_F = \frac{k}{(Ts+1)^2} = \frac{k}{T^2s^2 + 2Ts + 1},$$

where k - transfer coefficient and T - time constant.

The block scheme of the closed-loop system is presented in the Figure 1.

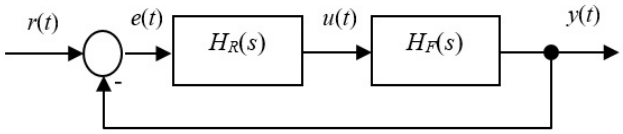

Figure 1. Structural scheme of the automatic control system.

For model estimation is proposed to be used P controller with transfer function:

$$H_R(s) = k_p,$$

where k_p is proportional tuning parameter.

The algorithm for closed-loop model identification is the following:

1. Implementation of the feedback control system with P controller.
2. Variation of the proportional tuning parameter $k_p > 0$ until the system achieves underdamped step response, as is presented in the Figure 2.
3. From obtained underdamped step response Figure 2 calculation the period of oscillations - T_0 :

$$T_0 = t_2 - t_1.$$

4. From Figure 2 calculation the value of the damping ratio [3]:

$$\xi = \frac{1}{\sqrt{1 + (\frac{2\pi}{\log d})^2}},$$

where d is decay ratio.

5. Calculation the value of natural frequency according to the relation [1-2]: ω_n

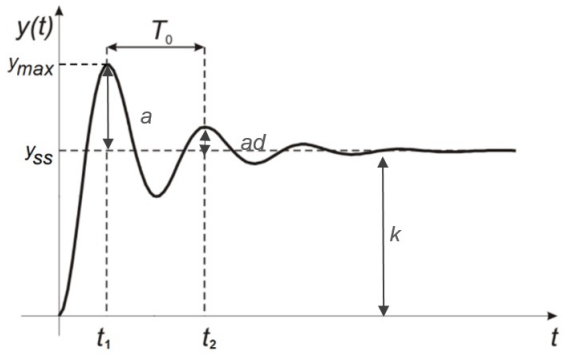

Figure 2. Underdamped step response of the closed-loop system.

6. Calculation of the $H_F(s)$ transfer function parameters:

$$T = \sqrt{\frac{k_p k + 1}{\omega_n^2}}.$$

Acknowledgement. *This paper was supported by the project 23.70105.5007.13T Research and synthesis of efficient attitude control algorithms for nanosatellites.*

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Water flow and erosion on vegetated surfaces. Part II: A coupled model

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Water flowing on the soil surface detaches and removes soil particles, giving rise to a fluid mixture composed by water and sediment grains. The fluid mixture and the soil can be both thought as two phases of a sole continuum medium separated by an interface - the soil surface - that changes as results of erosion and deposition. The presence of plants complicates the space configuration and the erosion process can be viewed as a phase transformation of the composite medium. The present work aims to provide a mathematical model for water flow and erosion that describes this phase transformation which includes changes in the spatial configuration and changes in the proportion of water and sediment grains in its internal structure. In this model, the flow is influenced by the suspended sediment and the soil surface may change over time.

On the Mathematics of Wave Phenomena

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Numerous and various mathematical treatments for wave phenomena have led to the unifying idea of interdisciplinary research, sometimes even surpassing experimental research. The analysis on particular fields of physics is found in the interpretative supremacy of quantum physics (correctly wave physics). Several mathematical aspects are presented in case studies (starting from solving the wave equation) in order to highlight the physical interpretation even in ideal cases (plane, spherical, cylindrical waves, etc.). The aspect of propagation in time and space (defining for waves) imposes the concept of phase and not frequency as the primary one; it justifies the usefulness of representations in the field of complex numbers. A general wave vision of reality, both concrete and abstract (for example in the formulation of boundary conditions or in numerical methods), is thus imposed. The philosophical implications are of great importance and the practical as well as the theoretical ones regarding oscillations are already covered by the theory of dynamical systems.

Keywords: oscillations, waves, propagation, physical interpretation

Sedimentation

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In the geological sciences, sedimentation is a process of deposition of a solid material from a state of suspension or solution in a fluid. This talk concerns the mathematical "pictures" of the geological process. A picture is a mathematical model that is inherently only partially faithful to the real process. Despite of their lack of fidelity, the mathematical models are widely used to obtain information, both quantitative and qualitative, about the morphology of the Earth's surface or lake bottom. Certain models are discussed and numerical results are also presented.

Mathematical modeling of the dense avalanche onset

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The starting point is the limit load problem of a shallow flow of a viscoplastic fluid/solid, with applications to the onset of dense avalanches with a small thickness on a basal surface with topography. The DVDS (Discontinuous Velocity Domain Splitting) method is used to reduce it to a shape optimisation problem (i.e. minimisation with respect to a subdomain). To solve the shape optimization problem, we introduce a numerical scheme based on a boundary variation method. Finally, we illustrate the proposed method with numerical simulations for academic and concrete problems using real geophysical data of the basal topography.
These results are obtained jointly with Ioan R. Ionescu (Paris).

Water flow and erosion on vegetated surfaces. Part I: A decoupled model.

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The present work aims to provide a mathematical tool to investigate the water-soil and water-plant interactions involved in the complex process of water flow on plant-covered soil surfaces. The mathematical model built here is based on an extended Saint-Venant system of equations for water flow and the Hairsine-Rose equations for soil erosion. We perform some numerical experiments in order to analyze the plant induced effects on water dynamics and soil erosion intensity and to observe if the model adequately reflects the essence of the reality. From a practical point of view, the tool described in this paper can help one to manage the environmental resources in order to avoid various water induced disasters.

Solutions to linear ODE systems corresponding to first-order chemical reactions and related inverse problems

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First-order chemical reactions always give rise to linear ODE systems, whose solutions are combinations of exponentials, trigonometric functions (sines and cosines), and polynomials [1].

However, not every function of this type is a solution to one of these problems; in other words, the inverse problem, in general, does not always have a solution.
This fact is fundamentally due to three theorems: the first one states that the zero eigenvalue always belongs to the spectrum of the associated matrix; the second expresses that the real part of the eigenvalues of the associated matrix are nonpositive; and the third states that the algebraic and geometric multiplicities of the zero eigenvalue always coincide [2].
These three results give the solutions certain characteristics and properties. Finally, related interesting inverse problems of existence, uniqueness and stability, are deeply studied [3].

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Modeling Thermochemical Phenomena by Using Saddle Point Results

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The aim of this paper is to obtain some results for certain saddle points involved in modeling specific thermochemical phenomena. The theoretical frame is based on conditional critical points on differential manifolds, where a central theorem is proven. Several applications are presented for simple thermochemical systems, also the interaction of two nonholonomic thermodynamic systems, the flow in mechanical disequilibrium. Moreover, the heat flow in thermal imbalance and the particle flow in chemical disequilibrium are studied. Such research can be applied to modeling heat transfer during phase transitions, solution formation during compound mixing and fate of chemical reactions.

Keywords: Saddle point; conditional critical points on manifolds; thermochemical phenomena; thermochemical interactions; chemical disequilibrium; heat flow.
2020 MSC : 35A15, 35J35, 35J50, 80A19, 80A30, 80A32

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Applicability of a Superlinear Problem with Odd Nonlinearity

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An appropriate variant of mountain pass theorem (MPT) is involved in developing a sequence of solutions for a certain superlinear Dirichlet problem with odd nonlinearities. Some results with abstract character are presented and correlated together with their links towards real world models. Based on this theoretical framework, a series of applications for concrete models such as multi-mode beams or condensate states, models for composite media (different strain or flux patterns), and metastable diffusion patterns for nonlocal transport are discussed. Applicability of the combination of odd and superlinear properties has been capitalized to prove multiplicity by symmetry, nonlinear Schrödinger standing waves (from nonlinear optics), quasilinear and fractional media (for materials, nonlocal diffusion), strongly indefinite systems (in coupled fields, spin or charge, waveguides). Moreover, the robustness beyond Ambrosetti-Rabinowitz type results has been analyzed as a basis for development and prospects of future research.

Keywords: mountain pass theorem; superlinear Dirichlet problem; odd nonlinearities; composite media; metastable diffusion patterns.
2020 MSC : 00A69, 35Q35, 35Q49, 47B93, 35A15, 35J35, 35J50, 35J60

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Recursive Combat Models

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The Lanchester combat model mathematical apparatus through which a combat action and several different perspectives on the same action can be obtained. This article presents a recursive combat model by [MacKay(2006)], which allows for the prediction of force ratios annihilation and combat times. The formulation and analysis of the Lanchester recursive combat model is performed for different sequential combat engagements, each time obtaining validation through a graphical approach.

Keywords: Lanchester combat models, recursive combat model, annihilation and combat times
2020 MSC : Primary 34C60, 34K60; Secondary 65L05, 65S05

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Effect of geometric shapes of nanoparticles on toxicity

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The toxicity of nanoparticles depends on their physical, chemical and other properties, size and geometric shape. Particle size, density, or interparticle spacing in nanoparticle assemblies depends strongly on the geometry of the template structure and spatial confinements. In nanoparticle design, the phenomenon that changes the existence and increase of magnetism in nanoparticles can be achieved by controlled geometric interpretation. Among the characteristic shapes of nanoparticles, two-dimensional (2D) and one-dimensional (1D) nanoparticle patterns on their surfaces are accepted for many applications and technological nanodevices. The geometric shapes of nanoparticles such as fibers, belts, wires, tubes, and spheres are very important in determining the toxicokinetics of nanoparticles, and the most worrisome shapes for toxicity are fibers or long wires. Because the most decisive factors in the toxicity of nanoparticles are their shape and length. The highly conflicting reports in toxicity studies of the same nanoparticle have been linked to their chemical, physical, and surface properties, particularly shape and size.

Keywords: geometric shape, nanoparticles, toxicity

Spatial Confinement of Different Types of Nanoparticles

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As the science of nanotechnology has developed, focus has shifted to nanoparticle assemblies rather than the functional and structural complexity of individual nanoparticles. Thus, combined systems provide quantitative and qualitative improvements in particle size, constituents, and physical or chemical properties. Spatial confinement results in increased efficiency and high resolution

properties of nanoparticles. The physical and/or chemical properties of nanoparticles are determined either by the type of motion granted to the electrons or by the area in which the electrons are confined under the influence of the forces they encounter. While the motion of unbounded electrons is not quantized, the motion of electrons bound to atoms and/or molecules is quantized because they are bound. The strength of confinement of electrons depends on the smallness of their field of action, with atomic confinement being the strongest form. For example, confining electrons to a large area in metal nanoparticles gives the metal its conductivity. When the electron movement within this metal nanoparticle is restricted by reducing its size, metal nanoparticles of different valences and shapes are formed. One of the two fundamental healing mechanisms of polymer nanocomposites is the spatial effect created by confining the polymer chains within a limited space between the nanoparticles.

Keywords: Spatial confinement, nanoparticles

Analytcal Description for the Working of the Pulsatory Liposome

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We consider a unilamellar liposome filled with an aqueous solution of osmotic solute inserted in a hypotonic aqueous medium. Due to the osmosis process the lipid vesicle swells up to a maximum size, when a transbilayer pore suddenly appears. The pore appearance changes the direction of the liposome evolution. Part of the internal solution leaks through this pore. The liposome deflates and returns to its initial size. The liposome swelling is described by a differential equation that has an analytical solution. Liposome deflation is described by three differential equations. This system of three differential equations can be solved numerically. In this work we obtain analytical solutions for the three differential equations that describe the relaxation stage of the pulsating liposome. These analytical solutions represent the functions that describe the time dependence of the liposome radius, the pore radius and the osmotic solute concentration.

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Control strategies for hydraulic servomechanisms of flight controls

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This paper outlines four key findings in hydraulic servomechanism design for flight controls. It revisits nonlinear stability theory, emphasizing the Lurie problem and Lifschitz condition. Impedance of servomechanism is another parameter which was shown to be crucial in preventing flutter of primary flight controls. Robust servomechanism synthesis methods are also considered, by including an anti-windup compensator. A fuzzy supervised neurocontrol is introduced for position tracking, combining neural and fuzzy logic to handle saturation. The paper ends with recent advances like backstepping control and delay compensation.
Acknowledgement. *This work was supported by grants of the Romanian Ministry of Research, Innovation, and Digitization, by NUCLEU Program project codes PN 23-17-02-03 and PN 23-17-07-01, Ctr. 36 N/12.01.2023.*

Keywords: mechano- and electrohydraulic servomechanisms; flight controls; Lurie problem and Lifschitz condition; servomechanism's impedance; anti-windup compensation; Davison's robust servomechanism problem; neuro-fuzzy synthesis; backstepping control; time-delay systems.

4. Real, Complex, Functional and Numerical Analysis

Some properties on translated and reverse topologies

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In the context of topological semilinear spaces (L, σ), one can naturally define 2 complementary topologies associated with σ - the translated topology and the reverse topology. It is shown by examples that these three topologies are distinct two by two. In the particular case of the Hausdorff upper topology on the family of non-empty subsets of a linear normed space, some classes of limits in the sense of reverse convergence are found, thus proving the consistency of the notion of reverse topology.

Applications of the differential subordination theory in univalence results involving fractional and convolution operators

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Differential subordination theory is a significant method for addressing the topic of obtaining geometric properties and univalence criteria of different types of operators. The bases of the differential subordination theory were established by Sanford S. Miller and Petru T. Mocanu in two papers published in 1978 and 1981. Many researchers have picked up the idea and enhanced the theory over the following decades. The techniques of this theory are applied in the investigation detailed in this presentation. Its applications concern establishing univalence criteria for operators defined by applying the fractional integral to Bessel function of the first kind and by using the convolution of Dziok-Srivastava linear hypergeometric operator and the Bessel function of the first kind, respectively.

Keywords: univalent function, convex function, fractional integral, differential subordination, best dominant, special functions. Subjclass201030C45, 30C80, 33C10.

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Deep learning methods and strategies for the enhancement of computed tomography image quality

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The most significant opportunity for improving computed tomography (CT) image quality and reducing patient radiation dose lies in noise reduction. In this field, in addition to conventional approaches, deep learning–based denoising methods are gaining increasing importance. A variety of network models exist, employing diverse training strategies (see in [1, 2]). In this presentation, we introduce several deep learning architectures using different supervised training schemes. For training, we used only mathematical phantoms with artificially simulated noise. We also present the noise model developed for a real multi-slice cone-beam CT system. The image quality improvements are demonstrated both on mathematical phantoms and on real CT scans.

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Certificate of positivity in Bernstein basis

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Let $P \in \mathbb{Z}[\mathbb{X}]$ be a polynomial of degree p with coefficients in the monomial basis of bit-size bounded by τ. If P is positive on $[-1, 1]$, we obtain a certificate of positivity (i.e. a description of P making obvious that it is positive) of bit-size $O(p^4(\tau + \log_2 p))$. Previous comparable results had a bit-size complexity exponential in p and τ.

Coupled systems driven by Leray-Lions type operators

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We are concerned with the weak solvability of nonlinear systems with variable exponents, involving Leray-Lions type operators. More precisely, we establish existence, uniqueness, and *a priori* estimates for the solution. One of our concerns was to relax the hypotheses and to find various examples of nonhomogeneous differential operators and of nonlinear terms which verify them. Consequently, we are able to bring numerous examples of systems obtained under our general hypotheses, such that our existence, uniqueness, and regularity results apply to all of them. In this manner, we show that it is possible to provide a unitary treatment for various systems.

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Riemann-Lebesgue integrability and applications in non-additive setting

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Starting from Riemann-Lebesgue integrability of a vector (real) function with respect to an arbitrary non-additive set function, we present some generalizations in two directions: the interval-valued setting, both for integrands and for set-functions, and to functions with values of real random variables. Some difficulties arise in these approaches:

-the techniques of the classical measure theory can no longer be used;
-the spaces are more difficult to use.

The motivation of it is mainly due to the fact that, though finite or countable additivity is a fundamental concept in measure theory, it can be useless in some modeling problems of decision making, data mining, economy, computer science, game theory, subjective evaluation, fuzzy logic, stochastic potential theory.

Homogenization of the diffusion-convection equation

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The paper presents the homogenization of a diffusion-convection problem posed on a periodically perforated domain. The method used is asymptotic development or the method of multiple scales. To demonstrate the convergence of the homogenization process, the double-scale convergence method was employed. The homogenized problem obtained is still a diffusion-convection problem in which the homogenized matrix and the homogenized convection vector retain the properties from the initial problem.

Hidden singularities in some nonlinear BVPs. Applications to lubrication approximation for thin viscous films

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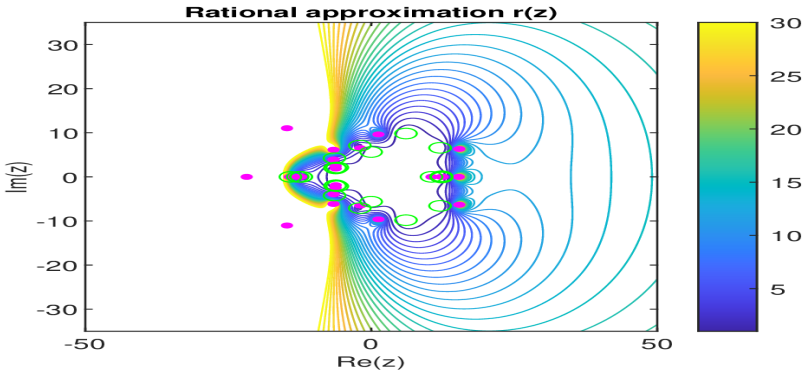
A thin viscous film flows on an inclined plane because of the competing effect of Marangoni and gravity stresses[1] The dimensionless nonlinear PDE for the thickness $h(x,t)$ of the film above the inclined plane, as a function of distance x down the plane and time t, that model this flow reads

$$h_t + \left(h^2 - h^3\right)_x + \left(h^3 h_{xxx}\right)_x + D\left(h^3 h_x\right)_x = 0,\ -\infty < x < \infty, \tag{1}$$

where $D = \left(\frac{9}{4}\frac{\tau^2}{\gamma\rho g}\right)^{1/3} \frac{\cot\alpha}{(\sin\alpha)^{1/3}}$, is a nondimensional parameter [2]
The equation will be solved along with some boundary conditions for travelling waves that connect an upstream height h_∞ to a (smaller) downstream height $b < h_\infty$. They read

$$h \to h_\infty,\ x \to -\infty \text{ and } h \to b > 0, x \to \infty. \tag{2}$$

The initial boundary value problem (1)-(2) is analysed using a three-step strategy. It consists of Chebyshev collocation, implemented as Chebfun, in conjunction with rational AAA interpolation and analytic continuation. By extending the solutions to these problems in the complex plane, we observe that the complex poles have no influence. On the other hand, the real ones involve singularities and indicate how long solutions can be extended through continuity.



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On collectively some sets of operators

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$L(X,Y)$ denotes the vector space of all bounded linear operators between normed spaces X and Y. A set $A \subseteq L(X,Y)$ is called collectively compact sets operators if $AB_X = \cup_{T\in A} T(B_X)$ is a relatively compact set in Y, where B_X is the closed unit ball in X. Let E be a normed vector lattice and let F be a normed space. An operator $T : E \to F$ is called a KB-operator if for every positive increasing normed bounded sequence (x_n) in E, there is an $x \in E$ such that (Tx_n) converges to Tx in F.

We investigate collective some sets of operators on normed vector lattices such as collectively KB-sets of operators, collectively quasi KB-sets of operators and so on. Also, we study the domination property of some collectively sets of operators on normed vector lattices.

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On a positive semigroup associated to a nonlinear evolution model in an ordered space II

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We consider a one-parameter C_0 semigroup of positive linear operators, arising in the formulation of a nonlinear evolution equation in an abstract state space in the sense of Davies, recently introduced in literature. In the present work, the study of the aforementioned semigroup, initiated in a previous paper, in the framework of an abstract state space, assumed to be also a Banach lattice, is extended to a non-lattice case. We provide an example showing that if we remove the lattice condition, then the above C_0 semi-group may become trivial. The main argument behind our example reveals that the norm entering into the definition of the abstract state space appears to be irrelevant to the construction of the example.

On the conditioning of Vandermonde matrices with mock-Chebyshev nodes

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The classical problem of polynomial interpolation on distinct nodes, when expressed in the monomial basis, gives rise to a Vandermonde matrix. However, a well-known drawback of the Vandermonde matrix is its tendency to become severely ill-conditioned when constructed from real nodes [1], even for moderate degrees. The severity of this ill-conditioning depends strongly on the distribution of the interpolation points and the choice of basis. Although the general recommendation is to employ nonuniform nodes that cluster near the endpoints, such as Chebyshev or Chebyshev–Lobatto points, in practice data are often available only at equally spaced locations. In such cases, interpolation becomes unreliable, suffering simultaneously from the Runge phenomenon and the ill-conditioning of the associated system. A practical remedy is to extract so-called mock-Chebyshev nodes from a dense uniform grid, thereby inheriting the advantageous properties of Chebyshev nodes [2, 3, 4].

In this study, we analyze the conditioning of Vandermonde matrices and demonstrate that employing mock-Chebyshev nodes yields a significant reduction in the condition number, with results comparable to those obtained using Chebyshev–Lobatto nodes.

Keywords: Vandermonde matrix, Condition numbers, Mock-Chebyshev nodes

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Generalized hyperbolic-type metrics on non-Euclidean metric spaces

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Hyperbolic metric plays a crucial role in both pure and applied mathematics, particularly in geometry, complex analysis, group theory and network theory. The lack of a counterpart of the hyperbolic metric in domains that are neither balls nor half-spaces, in higher dimensional Euclidean spaces, led to the construction of several hyperbolic-type metrics. We present recent advances on hyperbolic-type metrics defined in the complement of a proper closed subset M of an arbitrary metric space (X, d), that are generalizations of Gehring-Osgood metric, Vuorinen's distance ratio metric, the metric of Dovgoshey, Hariri and Vuorinen, the Nikolov-Andreev metric, the Ibragimov metric, the stabilizing metric of Boskoff and Suceavă, the $t-$metric of Rainio and Vuorinen. Gromov hyperbolicity, mutual bi-Lipschitz equivalence, metric completness and quasiconformal equivalence with the original metric are studied for the above-mentioned metrics. Also, if (X, d) is Ptolemaic and $s_{X\setminus M}(x, y) = \sup_{p\in M} \frac{d(x,y)}{d(x,p)+d(y,p)}$ for $x, y \in X \setminus M$ is the triangular ratio metric associated to M, we prove that $\log\left(1 + s_{X\setminus M}\right)$ is a metric that is strongly hyperbolic on $X \setminus M$ in the sense of Nica and Špakula.

Third-order fuzzy differential subordination and superordination results

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The concept of third-order fuzzy differential subordination emerged in 2024 as a result of the efforts made for extending the known knowledge related to second-order fuzzy differential subordination, established in two papers published in 2011 and 2012. The dual problem of the third-order fuzzy differential superordination has also been approached in 2024 by introducing the basic notions related to the concept of third-order fuzzy differential superordination. Applications of the two dual theories involving different operators have emerged in 2025 and they are the focus of this presentation. Sandwich-type theorems connecting the conclusions of the studies conducted using the particular methods of the theories of the third-order fuzzy differential subordination and superordination, respectively, are also presented

Keywords: fuzzy set, third-order fuzzy differential subordination, third-order fuzzy differential superordination, fuzzy dominant, fuzzy best dominant, fuzzy subordinant best fuzzy subordinant.
2010 MSC : 30C45, 30C80, 33C10.

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Recent studies on the fractional integral of the Gaussian hypergeometric functions

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After the fractional integral of Gaussian hypergeometric function was introduced and investigated using fuzzy differential subordination theory in 2022, it was used for obtaining new third-order differential subordination results in 2023. A new approach on its study concerns obtaining univalence conditions by applying the means specific to the classical differential subordination theory and the properties of the close-to-convex and Carathéodory functions. This approach was suggested by the recent developments on certain univalence conditions for the Gaussian hypergeometric function and it is the focus of this presentation.

Keywords: fractional integral, Gaussian hypergeometric function, starlike function, convex function, close-to-convex function, Carathéodory function, differential subordination, univalence condition.
2010 MSC : 30C80, 33C15, 30C45.

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Numerical Investigation of a Free Boundary Problem Involving Phase Chang

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Understanding phase change phenomena plays a vital role in scientific research, engineering design, and numerous industrial processes. In this study, we present a mathematical model for a two-phase melting process, where the thermal conductivity is temperature-dependent. The model is governed by a nonlinear parabolic partial differential equation defined in a moving domain or an unknown domain (Free boundary problem). Since an analytical solution to this problem is not feasible, a numerical investigation is carried out using a boundary-fixing explicit finite difference method. The proposed numerical scheme is shown to be consistent and conditionally stable. For a particular case of the model, the exact solution is derived, and the uniqueness of this solution is proven. A comparison between the numerical results and the exact solution for the specific case demonstrates excellent agreement. Additionally, the effects of various parameters on the phase change process are analysed.

Keywords: Free boundary problem, Phase change problem, Variable thermo-physical property, Finite difference method.

Numerical method for solving Blasius boundary layer model

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In this paper, we present a numerical method for solving the Blasius boundary layer model. The influence of transpiration on self-similar boundary layer flow over moving surfaces is analyzed using the modified operational matrix method. We derive the mathematical model and present theoretical results, including convergence analysis, error estimates, and the existence and uniqueness of the solution. Several numerical results are provided to demonstrate the efficiency of the proposed method.

Stieltjes differential problems with general conditions on the boundary

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We present an existence result for first order differential inclusions with very general boundary value conditions

$$\begin{cases} u_g'(t) \in F(t, u(t)),\ \mu_g-\text{a.e. } t \in [0,T] \\ L(u(0), u(T)) = 0 \end{cases}$$

involving the Stieltjes derivative ([1]) with respect to a left-continuous nondecreasing function $g : [0,T] \to \mathbb{R}$.
Here $F : [0,T] \times \mathbb{R} \to \mathcal{P}(\mathbb{R})$ is a Carathéodory multifunction and $L : \mathbb{R}^2 \to \mathbb{R}$ is a continuous map. The method consists in assuming the existence of appropriate notions of lower and upper solutions ([3]), with the help of which we prove the existence of solutions via a fixed point result for condensing mappings; several outcomes in literature can be deduced by particularizing the function g with respect to which the Stieltjes derivative is considered or by particularizing the map L describing the boundary condition.

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Estimations in Hölder spaces of some weak-singular integral operators

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Algorithmic implementation of collocation, quadratures, splin-collocation and splin-quadratures methods for solving weak-singular integral equations (WSIE)of second kind leads to necessity of concrete estimations in Hölder spaces of Fredholm and Volterra weak-singular integral operators (WSIO) that are used in these algorithms. We mention that only the existence of majority constants for the indicated WSIO was proved (see, for instance, the works [1] and [2]). This fact does not permit their application to numerical algorithms.

Denote by $H_\alpha[a,b]$ the Hölder space with exponent α $(0 < \alpha < 1)$ on $[a,b]$.

Theorem 1. *Let the function* $h(t,s) \in H_{\alpha,\alpha}[a,b]$ *and* $0 < \gamma < 1$. *Then, for any function* $\varphi(t) \in C[a,b]$, *we have*

$$G(t) = \int_a^b \frac{h(t,s)}{|t-s|^\gamma}\varphi(s)ds \in H_\theta[a,b],\ \theta = min(\alpha; 1-\gamma),$$

with Hölder's constant $c_1 = \|\varphi\|_c\Big(\|h\|_c\frac{2(2^\gamma+1)}{1-\gamma} + H^t(h;\alpha)\frac{2(b-a)^{1-\gamma}}{1-\gamma}\Big)$.

Let us define the functions

$$K_\rho(t,s) = \begin{cases} \frac{h(t,s)}{|t-s|^\gamma}, 0<\gamma<1; & |t-s| \geq \rho, \\ \frac{h(t,s)}{\rho^\gamma}; & |t-s| < \rho, \\ \rho << \min\{b-a;1\}. \end{cases}$$

Then for Fredholm WSIO, we prove

Theorem 2. *Assume that* $h(t,s) \in H_{\alpha,\alpha}[a,b]$; $\psi(t) \in C_\alpha[a,b]$; $0 < \gamma < 1$. *Then functions* $\chi_\rho(t) = \int\limits_a^b [\frac{h(t,s)}{|t-s|^\gamma} - K_\rho(t,s)]\cdot\psi(s)ds$, $\eta_\rho(t) = \int\limits_a^b K_\rho(t,s)\cdot\psi(s)ds$ *verify the following relations:*

$$|\chi_\rho(t)| \leq c_4\rho^{1-\gamma}\|\psi\|_c, \quad c_4 = \frac{2\gamma}{1-\gamma}\|h\|_c;$$

$$|\eta_\rho(t)| \leq (c_5 - c_4\rho^{1-\gamma})\|\psi\|_c, \quad c_5 = \frac{2(b-a)^{1-\gamma}}{1-\gamma}\|h\|_c;$$

$$\chi_\rho(t), \eta_\rho(t) \in H_\theta[a,b], \quad \theta = \min(\alpha; 1-\gamma).$$

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A Numerical Study for Systems of Fractional Differential Equations and Their Applications

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The operational matrix technique is a powerful method for addressing fractional initial value problems. In this study, we employ Block Pulse functions to approximate the solution. Traditionally, obtaining the expansion coefficients necessitates constructing operational matrices for integral, derivative, and product operators, which often results in a high-dimensional system of nonlinear algebraic equations. Solving such a system is computationally expensive, time-intensive, and can limit accuracy. To overcome these challenges, we introduce a modified approach that circumvents the need for solving a system of equations to determine the solution's coefficients. Instead, the coefficients are computed explicitly and iteratively based on previous values. We rigorously establish

the theoretical foundation of this approach, demonstrating that the iterative determination of coefficients leads to a sequence of functions that uniformly converges to the unique solution of the given system. Furthermore, we establish the existence and uniqueness of the solution. The proposed method is numerically validated through multiple examples, including two practical applications: one in optimal control theory and another related to the El Niño-Southern Oscillation (ENSO) system in global climate modeling. Various error measures, including the L_2-error and minimization error, are employed to assess accuracy. Comparative analysis with existing methodologies highlights that the proposed approach enhances accuracy, reduces computational costs, simplifies implementation, and improves efficiency over the conventional operational matrix method.

Set optimization via the embedding function

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The purpose of this communication is to see the efficiencies of set optimization as classical Pareto efficiencies for an associated function obtained by a Radstrom type embedding method in order to import in set optimization some techniques from vector optimization.

Sub-supersolution method for quasilinear Dirichlet problems with unbounded coefficient and intrinsic operator

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I want to use the sub-supersolution method in the case of a quasilinear Dirichlet problem which exhibits convection, with an intrinsic operator, and whose principal part contains an unbounded coefficient $G(u)$ depending on the solution u. A truncation technique leading to a priori estimates is developed not only for the reaction term in the equation, but also for the unbounded coefficient $G(u)$. The existence of a solution within the sub-supersolution interval is obtained together with uniform boundedness of the solution set.

A generalization theorem on quasi monotone sequences

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Recently, in [10] Bor has obtained two main theorems dealing with some applications of δ-quasi-monotone sequences to the factored infinite series and Fourier series. In this paper, we have generalized these theorems for $|A, \theta_n|_k$ summability method by using arbitrary triangle matrix instead of Riesz matrix. New and known results are also obtained for some absolute summability methods.

Keywords: Riesz mean, matrix summability, infinite series, Fourier series, Hölder inequality, Minkowski inequality.
2010 MSC : 26D15, 40D15, 40F05, 40G99, 42A24, 46A45.

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On absolute matrix generalizations of some factor theorems

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The aim of this paper is to generalize two theorems concerning weighted mean summability to $|A|_k$ summability by using quasi-f-power increasing sequences and an arbitrary triangular matrix instead of weighted mean matrix.

Keywords: Summability factors, absolute matrix summability, weighted mean summability, infinite series, Hölder and Minkowski inequalities.
2010 MSC : 26D15, 40C05, 40F05, 40G99.

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5. Probability Theory, Mathematical Statistics, Operations Research

On the degree of concordance with the discrete uniform distribution of pseudorandom number generators in the most commonly used computer applications

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In our work a statistical analysis of the degree of concordance with the uniform distribution in the discrete case of pseudorandom number generators was performed. The results obtained and the top of the most performing generators target the applications/programming languages Mathematica, Excel, Python, R, Random.org, C++, Java but also the decimals of the number PI. All of the tools related to Exploratory Analysis and Inferential Analysis of statistical data were used.

Keywords: Uniform distribution, pseudorandom number generator, Exploratory and Inferential Data Analysis.

A Framework for Sharing Staff between Outbound and Inbound Airport Processes

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This paper proposes an advanced simulation-optimization approach to evaluate and optimize the passenger flows within international airports. This approach allocates resources intelligently during the simulation process and balances demand and service quality. The resource allocation performed by our Advanced Resource Management (ARM) algorithm was used to develop an integrated system for arranging resources, identifying the proper resources, and allocating them throughout the model. It was used to investigate the influences of different staff allocation techniques on the inbound and outbound processes of an airport terminal. The purpose of the proposed simulation-optimization approach is to enhance passenger satisfaction through ensuring reasonable wait times during processing at the lowest cost possible (minimal staff hours).

Censored semi-Markov chains: modelling, estimation and smmR R package

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This presentation is concerned with parametric estimation of semi-Markov chains in the presence of several censoring schemes: censoring at the beginning and/or at the end of sample paths, no censoring, one or several trajectories. We present here the case of general semi-Markov chains, but also three other particular cases, that can be important from practical point of view, in some specific applications. The development of the associated R package, smmR, is also presented https://cran.r-project.org/web/packages/smmR/index.html

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Decoding EU Innovation in 2024: A Data-Driven Perspective Using EIS Indicators

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This study investigates the underlying structure of innovation performance in the European Union using the latest 2024 data from the European Innovation Scoreboard (EIS). Addressing gaps in prior research, this work combines traditional statistical techniques with modern machine learning-based feature selection methods to enhance the understanding of key innovation drivers across countries. Exploratory Factor Analysis (EFA), supported by the Kaiser–Meyer–Olkin (KMO) measure, Bartlett's test, and parallel analysis, reveals a robust unidimensional factor structure, explaining over 60% of the total variance. All indicators load substantially onto a single latent factor, confirming the internal coherence of the EIS construct. Reliability tests via Cronbach's alpha and McDonald's omega further validate this structure. This research offers an updated, data-driven perspective on innovation performance in the EU and demonstrates the relevance of

combining factor analysis with intelligent feature selection to uncover the most informative indicators of innovation across countries.

Keywords: European Innovation Scoreboard, Exploratory Factor Analysis, Reliability, Innovation.

Resilience for environment and resources using optimization models

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In this paper we present a model for a class of multiobjective control problems which can be use by logistics. The aim is to find the optimal solution, to carry maximum weight, and stll have the lower consumtion and lower emissions of carbon. The optimization model take into consideration the control multi-objective as the social, economic and ecological objectives to have triple benefits. We also present necessary definitions of efficient solution. In order to check optimality, we investigate restrictions associated to the form of Euler - Lagrange - type equations.

Solving extended informational matrix games in mixed strategies

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Consider the game in the strategic form $\Gamma = \langle X, Y, XAY^T \rangle$. All players know exactly the payoff matrices A and the sets of mixed strategies. Players maximize their payoffs. Consider also the following "informational decision making model": the player 1 knows exactly the value of the strategy chosen by the player 2, as well as, simultaneously, the player 2 knows exactly the value of the strategy chosen by the player 1. For solving such games we will apply the methodologies described in [1]. So the set of informational extended strategies of the player 1 which are presented like this $r : Y \to X$ and for all $y \in Y$, $r(y) = \|r_{ij}(y)\|_{i\in I}^{j\in J}$ where $r_{ij}(y)$ is the probability of choosing row i by player 1 when it is known that player 2 chooses column j with probability y_j. Corresponding, the set of informational extended strategies of the player which are presented like this $c : X \to Y$ and for all $x \in X$, $c(x) = \|c_{ij}(x)\|_{i\in I}^{j\in J}$ where $c_{ij}(x)$ is the probability of choosing column j by player 2 when it is known that player 1 chooses line i with probability x_j. We call these strategies "*mixed informational extended strategies*" and denote the sets of these strategies by $\boldsymbol{R} = \left\{ r : Y \to X \forall y \in Y, r(y) = \|r_{ij}(y)\|_{i\in I}^{j\in J} \right\}$, and $\boldsymbol{C} = \{c : X \to Y, \forall x \in X, c(x) = \|c_{ij}(x)\|_{i\in I}^{j\in J}\}$. It should be mentioned that the player 2 (corresponding player 1) do not know the informational extended strategies of the player 1 (corresponding player 2) and from this point of view we can consider that the game is in imperfect information structure over the sets of the informational extended strategies of the players. For any fixed strategies profile $(r, c) \in \boldsymbol{R} \times \boldsymbol{C}$ it is constructed the normal forms, generated by the informational extended strategies. So we obtain the following bimatrix games $\Gamma(r, c) = \left\langle X, Y, \sum_{j\in J} \sum_{i\in I} a_{ij} r_{ij} y_j, \sum_{i\in I} \sum_{j\in J} (-a_{ij}) c_{ij} x_i \right\rangle$ on the set of informational

non extended strategies. Here $\sum_{j\in J}\sum_{i\in I} a_{ij}r_{ij}y_j$ is the average value of the payoff of the player 1 when he knows, with probability r_{ij}, what values of $y \in Y$ will be choose by the player 2. Similarly, $\sum_{i\in I}\sum_{j\in J}(-a_{ij})c_{ij}x_i$ is the average value of the payoff of the player 2 when he knows, with probability c_{ij}, what values of $x \in X$ will be choose by the player 1.

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Infinite-horizon singular H_∞ control problem

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We consider an infinite horizon H_∞ control problem for linear systems with disturbances. The case of a singular weight matrix for the control cost in the cost functional is treated. In such a case, a part of the control coordinates is singular, meaning that the H_∞ control problem itself is singular. For solving this problem we associate the original singular problem with a new H_∞ control problem for the same equation of dynamics. The cost functional in the new problem is the sum of the original cost functional and an infinite horizon integral of the squares of the singular control coordinates with a small positive weight. This new H_∞ control problem is regular, and it is a partial cheap control problem. Based on an asymptotic analysis of this partial cheap control problem, a controller solving the original singular control problem is designed.

Keywords: H_∞ Control Problem, Riccati Matrix Algebraic Equation, Asymptotic Design of Controller

Total Alert Time of the Stochastic Systems with Multiple Final Sequences of States

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The total alert time was defined in [2], for stochastic systems with final critical state of order m and independent transitions, as the cumulative time during which the system remains in its critical state throughout its evolution. It was proved that the total alert time can be expressed in terms of evolution time of the system with the same critical state of order $m-1$.
Next, we extend the total alert time for the systems with multiple final sequences of states. A state is critical at given moment of time if it is part of one final sequence of states and the system has just consecutively passed through all previous states from that sequence. The total alert time represents the cumulative time during which the state of the system is critical.

The methods from [1] reduce the complexity by decreasing the length of the final sequences of states to 1 and by eliminating the dependency of criticality on time context. The total alert time can be handled as the visiting time of the set of critical-related states in the new system.
The states, starting from which any critical-related state is not reachable, can be ignored, since the probability of visiting time, starting from these states, is zero. For the remaining states, the distribution of the visiting time (split by initial state) can be written as a recurrence of the form $P(n) = Q \cdot P(n) + R \cdot P(n-1)$, where Q is a weakly chained substochastic matrix. According to [3], this property of matrix Q ensures that the matrix $I - Q$ is invertible and, as consequence, the distribution of the total alert time is a homogeneous linear recurrence over $\mathbb{R}$. This allows us to use the existing algorithms for probabilistic characterization of the total alert time.

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First-passage time distributions of a Brownian motion observed after a random delay

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Let $\{W(t), t \geq 0\}$ be a Brownian motion starting at $W(0) = 0$, with drift parameter $\mu > 0$ and variance parameter σ^2. The probability density function (p.d.f.) of the first-passage time

$$T_d := \inf\{t > 0 : W(t) = d > 0\} \tag{1}$$

is well known to be given by

$$f_{T_d}(t) = \frac{d}{\sqrt{2\pi\sigma^2 t^3}} \exp\left\{-\frac{(d-\mu t)^2}{2\sigma^2 t}\right\} \quad \text{for } t > 0. \tag{2}$$

This density function is a particular *inverse Gaussian distribution*.

Now, suppose that the stochastic process can only be observed after a continuous random time τ, and let T be the time required to record the first visit to the threshold d, so that

$$T := \tau + T_d. \tag{3}$$

Moreover, if we assume that the random variables τ and T_d are independent, then we can write that the p.d.f. of T is the convolution product of the p.d.f. of τ with that of T_d:

$$f_T(t) = f_\tau(t) * f_{T_d}(t) := \int_0^t f_\tau(s) * f_{T_d}(t-s)\,\mathrm{d}s = \int_0^t f_\tau(t-s) * f_{T_d}(s)\,\mathrm{d}s. \tag{4}$$

p Schwarz (2002) treated the case when τ has an exponential distribution with parameter λ:

$$f_\tau(t) = \lambda e^{-\lambda t} \quad \text{for } t \geq 0. \tag{5}$$

In this talk, we will study the case when τ has an Erlang distribution. Then, we will show how to approximate the p.d.f. of T using its moments and orthogonal polynomials

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Nash Equilibria for Stochastic Positional Games with Average and Discounted Payoffs

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Stochastic games where introduced by Shepley [1], whose original formulation later has been extended by other for a more large class of problems of game theory. Shapley considered a two-player zero-sum stochastic game with a discounted payoff criterion and proved the existence of the value the game. Fink [2] extended the results of Shepley for n-player stochastic games with discounted payoffs for the players. Gillette [3] introduced limiting average payoffs for stochastic games with finite state and action spaces for which Mertens and Neyman [4] proved the existence of so-called ϵ-Nash equilibrium for two-player stochastic games with average payoffs. In general, the problem of the existence of Nash equilibria for stochastic games with average payoffs is a difficult problem and until now it remains an open problem. In this presentation we consider a class of stochastic games with average and discounted payoffs for which Nash equilibria exist in stationary strategies. In a stochastic positional game the set of states is divided into several disjoint subsets, where each subset is regarded as a position set for one of the players and each player control the Markov process only in his position set. We call this class of games *stochastic positional games*. We show that for an arbitrary stochastic positional game there exists a Nash equilibrium in mixed stationary strategies and for a stochastic positional game with discounted payoffs there exists a Nash equilibrium in pure stationary strategies. Additionally, we show that for a zero-sum stochastic positional game of two players with average and discounted payoffs there exists a Nash equilibrium in pure stationary strategies. Some of these results can be found in [5].

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When the coalescence of two graphs is a König-Egerváry graph?

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Let $\alpha(G)$ denote the cardinality of a maximum independent set, while $\mu(G)$ be the size of a maximum matching in the graph $G = (V(G), E(G))$. If $\alpha(G) + \mu(G) = |V(G)|$, then G is a *König-Egerváry graph* [2, 3, 4].

Let G_1 and G_2 be two vertex-disjoint graphs. The graph $(G_1 \div G_2)(v, u : x)$, obtained from G_1 and G_2 by identifying $v \in V(G_1)$ and $u \in V(G_2)$ in a vertex labelled x, is the *coalescence* of G_1 and G_2 via v and u [1].

The presentation focuses on necessary and/or sufficient conditions for a coalescence of two graphs to be König-Egerváry**.** In addition, we highlight the properties of μ-critical and α-critical vertices within coalescences.

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A greedy algorithm for 2-restricted optimal pebbling problem

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The graph pebbling problem involves the distribution and movement of indistinguishable pebbles on the vertices of a connected graph. A pebbling move consists of removing two pebbles from a vertex and placing one pebble on an adjacent vertex. A pebble distribution is considered *solvable* if it is possible to move at least one pebble to any vertex with a sequence of pebbling moves. A fundamental question in graph pebbling is to determine the minimum number of pebbles required to guarantee that any target vertex can be reached by a sequence of pebbling moves, regardless of the initial pebble distribution. This minimum number is known as the *pebbling number* of the graph, denoted by $\pi(G)$ [2, 3].

Variations of the standard pebbling problem have been introduced and studied [1]. One such variation is *k-restricted optimal pebbling*, where the limit of at most k pebbles per vertex applies only to the initial distribution. This paper focuses on the specific case of $k = 2$, known as *2-restricted optimal pebbling* denoted by $\pi_2^*(G)$. This is an NP-complete problem [4], so we developed a greedy algorithm that provides an approximate solution.

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New formulas for skewness and kurtosis based dynamic range for gray level and color images

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The dynamic range of a gray level or a color image plays an important role in analysis and processing methods. An important method for determining the dynamic range is the one in which we use statistical measures for the histograms associated with the components of an image.

Let us consider the discrete probability distribution P:

For $i = 1, 2, \ldots, n$

$$P(x_i) = p_i \tag{1}$$

where $(x_1, \ldots, x_n) \in (-\infty, \infty)^n$, $(p_1, \ldots, p_n) \in [0, 1]^n$ and

$$\sum_{i=1}^{n} p_i = 1 \tag{2}$$

To calculate the dynamic range we will use the following statistical moments defined for the distribution P:

$$\mu(P) = \sum_{i=1}^{n} p_i x_i \tag{3}$$

$$\delta(P) = \sum_{i=1}^{n} p_i \left| x_i - \mu \right| \tag{4}$$

$$\lambda(P) = \sum_{i=1}^{n} p_i \left| x_i - \mu \right| (x_i - \mu) \tag{5}$$

$$\sigma^2(P) = \sum_{i=1}^{n} p_i \left(x_i - \mu\right)^2 \tag{6}$$

$$\eta(P) = \sum_{i=1}^{n} p_i \left(x_i - \mu\right)^3 \tag{7}$$

$$\omega(P) = \sum_{i=1}^{n} p_i \left|x_i - \mu\right|^3 \tag{8}$$

$$\rho(P) = \sum_{i=1}^{n} p_i \left(x_i - \mu\right)^4 \tag{9}$$

Two of the most important statistical measures derived from the statistical moments defined above are skewness and kurtosis. Pearson defined the following measures for skewness and kurtosis [4], [5], [6] :

$$\gamma_p(P) = \frac{\eta(P)}{\sigma^3(P)} \tag{10}$$

$$\kappa_p(P) = \frac{\rho(P)}{\sigma^4(P)} \tag{11}$$

while Geary defined the following measures for skewness and kurtosis [2], [3]:

$$\gamma_g(P) = \frac{\lambda(P)}{\sigma^2(P)} \tag{12}$$

$$\kappa_g(P) = \frac{\delta(P)}{\sigma(P)} \tag{13}$$

In this paper we will use the following formulas for skewness and kurtosis:

$$\gamma_I(P) = \frac{\lambda(P)}{\delta^2(P)} \tag{14}$$

$$\gamma_{II}(P) = \frac{\eta(P)}{\delta^3(P)} \tag{15}$$

$$\kappa_I(P) = \frac{\sigma^2(P)}{\delta^2(P)} \tag{16}$$

$$\kappa_{II}(P) = \frac{\omega(P)}{\delta^3(P)} \tag{17}$$

$$\kappa_{III}(P) = \frac{\omega(P)}{\sigma^3(P)} \tag{18}$$

Next, using the two-point approximation method of a probability distribution similar to that proposed by Rosenblueth [7] or the three-point approximation method of a probability distribution similar to that proposed by Gorman [1], we will obtain the dynamic range for gray level or color images. Thus, if $Q_2 = \{(p_1, v_1), (p_2, v_2)\}$ is the distribution obtained by the two-point approximation or $Q_3 = \{(p_1, v_1), (p_0, \mu), (p_2, v_2)\}$ is the distribution obtained by the three-point approximation, the dynamic range is defined by

$$\Delta = [v_1, v_2] \tag{19}$$

where for both Q_2 and Q_3 we assumed that $v_2 > v_1$.

Finally, using formulas presented above, we will construct transformation functions to improve the quality of both gray level and color images.

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Data analysis using low-cost air quality sensors for outdoor monitoring in urban areas

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Urban areas are experiencing increased levels of air pollutants due to various sources, but the traffic has an important contribution, being present everywhere. Governmental and local authorities are using fixed and mobile stations for monitoring the concentrations in multiple locations, still, their number is not always sufficient to assess these at bigger distances from the monitoring points. Instrumentation based on reference methods does capture the proper concentrations but the monitors or sensors are expensive and their maintenance is difficult and costly. Furthermore, some reference methods do not provide real time measurements. Recently, low cost sensors for air pollution have gained attention due to their versatility, low operating costs, the possibility to increase the area with measurements eliminating ¡93¿blind¡94¿ spots or interpolated data with a degree of uncertainty. These sensors were integrated with satellite information, AI and statistical algorithms providing a powerful monitoring system with increased reliability. In this matter, the EARTHSENSE Zephyr® (https://www.earthsense.co.uk/zephyr) is an award-winning, certified, indicative ambient air quality monitor that accurately measures harmful gases and particulate matter in real-time to help identify pollution hotspots at a localized level such as busy road junctions or the impact on sensitive areas (schools, hospitals, kindergartens). Extensive networks can be easily deployed for city-wide pollution data used to aid the development of smarter and cleaner

towns and cities. The data provided requires preliminary analysis for correlation with meteorological data, spatial data by applying techniques to identify trends and patterns in air quality data, often using statistical models like ARIMA for forecasting. Time series analysis (TSA) is an important tool of statistics providing suitable indicators for time-dependent data including the possibility of time series forecasting. Some of the commonly used techniques are random moving averages, random walks, trend models, seasonal exponential smoothing, Boltzmann composite lattice, and autoregressive parametric models and its variants. In this work, the data analysis comprised the exploration of the time series recorded by the Zephyr system in the campus of Valahia University, in Targoviste city using correlation analysis. ARIMA and SARIMA. ARIMA is a parametric model relating the most recent data value to the previous data values and previous noise gives the best forecast for future data. SARIMA is an extension to ARIMA, which supports the direct modeling of the seasonal component of the univariate data series. The results are expected to optimize some operational features of the system in view of increasing the network capabilities for air quality monitoring.

Keywords: particulate matter, ozone, meteorological parameters, Time Series Analysis, ARIMA, SARIMA
Acknowledgement. *This research work was carried out with the support of the CNFIS-FDI-2025-F-0421 project funded by the National Commission for Higher Education Financing (CNFIS), Ministry of Education and Research from Romania.*

Projective Shape Analysis for Spatial Orientation in Virtual Environments

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This talk is based on the joint work with Alexander Garthe DZNE (Deutsches Zentrum für Neurodegenerative Erkrankungen, Dresden, Germany), Victor Patrangenaru (Florida State University, USA) and Robert Paige (Missouri S & T, USA). We introduce and develop a projective shape analysis for the study of cognitive abilities evaluated based on learning behaviour in the DSNT (Dresden Spatial Navigation Task) virtual navigational experiment ([1]). DSNT adapts the classical water maze test for humans, and was developed at DZNE (The Research Institute for Neurodegenerative Diseases from Dresden, Germany). This new mathematical modelling of the spatial orientation and learning is based on recent concepts in object-oriented data analysis like extrinsic covariance and extrinsic cross-covariance as well as novel statistical testing methods for random objects on manifolds ([2]). Additionally, new numerical algorithms will be developed, studied and finally implemented in an open-source mathematical software like R and will be used to evaluate our conclusions and to present the data visually.

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Minmax and Maxmin Modified Discrete Uniform Distributions as Lifetime Distributions in Networks Reliability

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In our paper, two new probability distributions are proposed that model, in particular, the lifetime of serial-parallel and parallel-serial networks. The corresponding survival/reliability functions, as well as the mortality/hazard intensity functions, are derived.
The problem of finding maximum likelihood estimators for the lifetime parameter of the network units in question based on uncensored and censored statistical data regarding the lifetime of the modeled networks is addressed and solved. The results are illustrated by appropriate examples.

Keywords: Uniform distribution, serial-parallel and parallel-serial networks, lifetime distribution, reliability function, maximum likelihood estimators.

One efficient method for solving the linear multicriteria optimization model in integers

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Currently, multi-criteria optimization models are in increasing demand. This is due to the fact that numerous problems in the most diverse practical fields require solutions that are acceptable from the point of view of several criteria. A wide range of practical optimization problems in various fields lead to the solution of mult-icriteria linear optimization models [1] in integers. In this paper we propose one efficient interactive method for solving the next linear multi-criteria model in integer:

$$\begin{cases} \left\{ \begin{array}{c} \min \\ \max \\ x \in D \end{array} \right\} F_k(x) = \sum_{j=1}^{n} c_{kj} x_j, \quad k = \overline{1,r} \\ A \cdot x \leq b \\ x \in Z^+ \end{cases} \tag{1}$$

in which: $D = \left\{ x = (x_1, x_2, \ldots, x_n)^T \,|Ax \leq b, x \in Z^+ \right\}$, $A = \|a_{ij}\|$ is an array of size $m \times n (m < n)$, $C = \|c_{kj}\|$, is an array of size $r \times n (r < n)$, x is a n-dimensional column vector and b is a m -dimensional column vector.

The paper proposes the interactive global utility maximization method [2] for solving the model (1). According on the utilities assigned by the decision maker, the proposed algorithm generates sets of efficient solutions for the model, considering all possible combinations of the optimal or worst values of the criteria in R^+ or in Z^+, which maximize the following objective function of their utility:

$$FU_j = \alpha_j F_j(X) + \beta_j, \; j = \overline{1,r}. \tag{2}$$

The efficiency of the method can be appreciated through concrete decision-making situations. The theoretical justifications of the method, as well as an example solved using this method, are provided in the full study.

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Robust Estimations and Tests for Moment Condition Models

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We present robust minimum empirical divergence estimators for moment condition models, based on truncated orthogonality functions and dual form of divergences. The class of these estimators is indexed by the phi-function corresponding to the used divergence and contains some known estimators as special cases. The robust exponential tilted estimator can be obtained through our procedure when using the Kullback-Leibler divergence, also a robust version of the empirical likelihood estimator is obtained in case of using the modified Kullback-Leibler divergence. For the class of these estimators we proved robustness properties using the influence function approach, as well as asymptotic properties including consistency and asymptotic laws. Robust tests based on estimators of the divergence are also derived. Some examples using Monte Carlo simulations illustrate the performance of the proposed methods.

How common sense can be misleading in ruin theory

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Some graphical representations of ruin probability, computed mainly for Erlang-type claims, suggest an idea that intuitively seems to be true: if the first claims are small, then the chance of being ruined is also small. However, for other claims, this does not hold, as is shown by counterexamples.

Keywords: Ruin probability, nonhomogeneous claims, stochastic order.

6. Algebra, Logic, Geometry (with applications)

Ulrich bundles on surfaces in the projective space

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The study of Ulrich bundles is a subject of active research in the last years. After a short introduction to the subject, I will speak about the case of low degree surfaces in the projective 3-space. In particular, we shall explore the posibility to apply the Cayley-Bacharach theorem to the construction of low rank Ulrich bundles on surfaces of degree $9 \leq d \leq 15$ in the projective space.

On the spectrum of conharmonic curvature tensor of six-dimensional Kählerian submanifolds in Cayley algebra

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Conharmonic transformations are conformal transformations that preserve the property of harmonicity of smooth functions. This type of transformations was introduced into consideration by the Japanese mathematician Y. Ishii [1]. It is known that such transformations have a tensor invariant — the so-called conharmonic curvature tensor. Note that complementing of a Riemannian structure to an almost Hermitian structure allows us to single out some additional conharmonic invariants [2].

In the present work, we consider the conharmonic curvature tensor of 6-dimensional Kählerian submanifolds in the octave algebra. The Kählerian (and in the general case, almost Hermitian) structure on such submanifolds is induced by the so-called Gray-Brown 3-fold vector cross products in Cayley algebra [3].

As the main result we mark out the calculation of the so-called spectrum of the conharmonic curvature tensor for an arbitrary 6-dimensional Kählerian submanifold in the octave algebra. By the concept of the spectrum of a tensor [2], we mean the minimal set of the components on the space of the associated G-structure that completely determines this tensor.

Theorem. *The spectrum of conharmonic curvature tensor of a 6-dimensional Kählerian submanifold in Cayley algebra is defined by the following equalities:*

$$Ch_{abcd} = 0; \quad Ch_{\hat{a}bcd} = 0;$$

$$Ch_{\hat{a}\hat{b}cd} = -\frac{1}{2}\left(T^7_{\hat{a}\hat{h}}\,T^7_{hc}\,\delta^b_d + T^7_{\hat{b}\hat{h}}\,T^7_{hd}\,\delta^a_c - T^7_{\hat{a}\hat{h}}\,T^7_{hd}\,\delta^b_c - T^7_{\hat{b}\hat{h}}\,T^7_{hc}\,\delta^a_d\right);$$

$$Ch_{\hat{a}b\hat{c}d} = -\,2\,T^7_{\hat{a}\hat{c}}\,T^7_{bd} + \frac{1}{2}\left(T^7_{\hat{a}\hat{h}}\,T^7_{hd}\,\delta^c_b + T^7_{\hat{c}\hat{h}}\,T^7_{hb}\,\delta^a_d\right).$$

Here $\left\{T^{\varphi}_{kj}\right\}$ are the components of the configuration tensor, or of the second fundamental form of the immersion of a 6-dimensional Kählerian submanifold into Cayley algebra; $\varphi = 7,\ 8$; $a,\ b,\ c,\ d,\ h = 1,\ 2,\ 3$; $\hat{a} = a + 3$; $k,\ j = 1,\ 2,\ 3,\ 4,\ 5,\ 6$.

It is clear that the calculated components of the conharmonic curvature tensor allow us to study the so-called conharmonic analogues of the Gray¡92¿s identities from [4]. Such analogues were introduced into consideration by V.F. Kirichenko, A. Rustanov and A. Shihab in [5]. Another possible application of the obtained results is the further development of the created by A. Cray and V.F. Kirichenko theory of 6-dimensional almost Hermitian submanifolds in the octave algebra (see the surveys [6], [7], etc).

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On the curvartures of the Kähler golden manifolds

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In this paper we deal with the investigation of the curvature tensors of a Kähler golden manifold. We study some properties of the sectional curvature of a Kähler golden manifold and we prove that, a Kähler golden manifold which has a constant sectional curvature is flat, under certain conditions. We obtain the form of the Riemannian curvature of a Kähler golden manifold, which has constant Φ-holomorphic sectional curvature and we obtain some relations between Φ-holomorphic bi-sectional curvature and Golden sectional curvature.

Keywords: Almost complex golden structure; Kähler golden manifold; sectional curvature; holomorphic sectional curvature
2010 MSC : 53B20; 53B25; 53C42; 53C15

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Some Open Problems in the Theory of Topological Quasigroups and Groupoids

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A non-empty set G is said to be a *groupoid* with respect to a binary operation denoted by $\{\cdot\}$, if for every ordered pair (a, b) of elements of G there is a unique element $ab \in G$.
A quasigroup is a binary algebraic structure in which one-sided multiplication is a bijection in that all equations of the form $ax = b$ and $ya = b$ have unique solutions [1].
A groupoid G is called a primitive groupoid with divisions, if there exist two binary operation $l : G \times G \to G$, $r : G \times G \to G$ such that $l(a, b) \cdot a = b, a \cdot r(a, b) = b$ for all $a, b \in G$. Thus a primitive groupoid with divisions is a universal algebra with three binary operations.
A primitive groupoid G with divisions is called a quasigroup if the equations $ax = b$ and $ya = b$ have unique solutions. In a quasigroup G the divisions l, r are unique. If the multiplication operation in a quasigroup $(G, \cdot)$ with a topology is continuous, then G is called a semitopoligical quasigroup. If in a semitopological quasigroup G the divisions l and r are continuous, then G is called a topological quasigroup.
A groupoid $(G, \cdot)$ is called:
-*medial* if $xy \cdot zt = xz \cdot yt$ for all $x, y, z, t \in G$.
-*paramedial* if $xy \cdot zt = ty \cdot zx$ for all $x, y, z, t \in G$.
-*AD-groupoid* if $a \cdot bc = c \cdot ba$ for all $a, b, c \in G$.
-*AG-groupoid* if $ab \cdot c = cb \cdot a$ for all $a, b, c \in G$.
-*Manin groupoid* if $x(y \cdot xz) = (xx \cdot y)z$ for all $x, y, z \in G$.
-*Cote groupoid* if $x(xy \cdot z) = (z \cdot xx)y$ for all $x, y, z \in G$.
While if an *AG-groupoid* $(G, \cdot)$ satisfying the identity $a \cdot (b \cdot c) = b \cdot (a \cdot c)$ for all $a, b, c \in G$ is called *AG^{**}-groupoid*.
We define a *Ward groupoid* as any groupoid $(G, \cdot)$ containing an element $e \in G$ such that $a^2 = a \cdot a = e$ and $(ab) \cdot c = a \cdot (c \cdot (e \cdot b))$, for all $a, b, c \in G$. A groupoid $(G, \cdot)$ is called a *Schröder Second Law groupoid* if it satisfies the law $(ab) \cdot (ba) = a$ for all $a, b \in G$ [6]. The identity $(ab) \cdot (ba) = b$ for all $a, b \in G$ is known as *Stein's Third Law* [6]. The concept of (n, m)-identities was introduced by M.M. Choban and L.L. Chiriac in [2].
We study the problems formulated below.
Problem 1. Let G be an *binary groupoid*. Under which conditions G with a locally compact Hausdorff topology can be "transformed" into a topological quasigroup?
Robert Ellis, in 1957, proved that a group with a locally compact Hausdorff topology making all translations continuous also has jointly continuous multiplication and continuous inversion, and is thus a topological group.
We examine a similar problem for quasigroup structure. We extend the theorem of R.Ellis to the case of *AD*- groupoids, which satisfies certain conditions.
Quasigroup $(G, \cdot)$ is a $T - quasigroup$ if and only if there exist an abelian group $(G, +)$, its automorphisms φ and ψ, and a fixed element $a \in G$ such that $x \cdot y = \varphi(x) + \psi(y) + a$ for all $x, y \in G$.
Problem 2. Let Q be a $T - quasigroup$. Under which conditions the Q is a quasigroup (of its T - forms $(Q(+), \varphi, \psi, a)$) satisfying the identities P_i, where $i = 1, 2, ..., k$?
The following result has been obtained:

Theorem 1. *Let G be a $T-$quasigroup. Then G is $AG-$quasigroup if and only if any for of its $T-forms$ $(Q(+),\varphi,\psi)$ is $\varphi^2(x)=\psi(x)$.*

Problem 3. Under which conditions the binary topological groupoid with the algebraic properties $P_1,P_2,...,P_k$ can be "transformed" into a topological quasigroup with the algebraic properties $P_1,P_2,...,P_k$?
We give some results for Schröder, Stein, medial, AG and AD-topological quasigroups.
Problem 4. Let $(G,\cdot)$ be a topological quasigroup obeying the algebraic properties P_i, where $i=1,2,...,k$, with an (n,m)-identity e. Under which conditions an open compact neighborhood P, such that $e\in P$, contains an open compact subquasigroup $(Q,\cdot)$ with properties P_i and (n,m)-identity of $(G,\cdot)$.
The following theorems have been proven.

Theorem 2. *Let $(G,\cdot)$ be a $Ward$ topological quasigroup with a $(2,1)$-identity e and $x^2=e$ for every $x\in G$. If P is an open compact neighborhood such that $e\in P$, then P contains an open compact $Ward$ subquasigroup $(Q,\cdot)$ with a $(2,1)$-identity of $(G,\cdot)$.*

Theorem 3. *Let $(G,\cdot)$ be a topological AG^{**}-quasigroup with an $(1,2)$-identity e and $x^2=e$ for every $x\in G$. If P is an open compact neighborhood such that $e\in P$, then P contains an open compact AG^{**}-subquasigroup $(Q,\cdot)$ with an $(1,2)$-identity of $(G,\cdot)$.*

In the context of topological groups an analogous result appears in the work of Pontrjagin ([7], Theorem 16).
Problem 5. Let $(G,+,\tau)$ be a commutative topological group. Under which conditions on the set $G\times G$ can be defined the binary operation $(\circ)$ such that, $(G\times G,\circ,\tau_G)$, relative to the product topology τ_G, is a nonassociative topological quasigroups obeying certain laws?
We give a new method of constructing non-associative topological quasigroups obeying certain laws: medial, semimedial, paramedial, bicommutative, Manin, Cote and GA non-associative topological quasigroup.
The results established here are related to the work in ([3,4,5]).

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On a prolongation of quasigroups using two transversals

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A groupoid $(Q,\cdot)$ is called a quasigroup if, for every $a,b \in Q$, the equations $a \cdot x = b$ and $y \cdot a = b$ have unique solutions in Q. The recursive derivative of order k of a quasigroup $(Q,\cdot)$, denoted by $(\overset{k}{\cdot})$, where k is a natural number, is defined as follows: $x \overset{0}{\cdot} y = x \cdot y$, $x \overset{1}{\cdot} y = y \cdot (x \cdot y)$, $x \overset{k}{\cdot} y = (x \overset{k-2}{\cdot} y) \cdot (x \overset{k-1}{\cdot} y), \forall k \geq 2, \forall x, y \in Q$.
A quasigroup $(Q,\cdot)$ is called recursively r -differentiable if its recursive derivatives of order k are quasigroup operations, for all $k = 1, \ldots, r$.
A prolongation of a finite quasigroup is a process of extending the quasigroup by adding one or more new elements and redefining the operation to obtain a new quasigroup of higher order. The notion of prolongation was introduced by Belousov in 1967, although the construction of quasigroups prolongations was first studied by Bruck in 1944, who considered finite idempotent quasigroups for this purpose [1]. Later, some other methods of quasigroups prolongations have been proposed and studied, many of which use transversals [2-5].
A transversal of a Latin square of order q is a set of q cells taken by one from each row and each column, in which the elements are pairwise different. In finite quasigroups the transversals are defined by complete mappings. A free transversal of a Latin square L of order q is a set of q cells taken by one from each row and each column of L.
We propose a method of quasigroups prolongation using two transversals that intersect exactly in one cell, and study the recursive 1-differentiability of such prolongations in the present work. Remark that there does not exist Latin squares of order 3 with 2 transversals that intersect exactly in one cell.

Proposition 1. *A Latin square of order 4 has exactly 24 free transversals. Moreover, there are 96 pairs of free transversals that intersect exactly in one cell.*

Theorem 1. *There exist exactly 48 different Latin squares of order 5 with two transversals intersecting exactly in one cell, where one transversal is on the main diagonal with a fixed order of elements. No recursively 1-differentiable prolongation exists for quasigroups of order 5 constructed using the main diagonal with one of the fixed order of elements* $\{1,2,3,4,5\}$ *or* $\{2,3,4,5,1\}$*, and any other transversal that intersect it exactly in one cell.*

Acknowledgement. *The Institutional Research Program of the Moldova State University for* $2024-2027$ *years, subprograms* 011302 *"MANSDP" and* 011303 *"SATGET", has supported part of the research for this work.*

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On Manifolds with Fundamental Group Generated by Hyperbolic Translations

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A method of construction of countable series of three-dimensional polyhedra that decompose normally and regularly the hyperbolic space is proposed. Hyperbolic 3-manifolds with totally geodesic boundaries are constructed by identifying pairs of hyperfaces only by hyperbolic translations. These hyperbolic manifolds have as their boundaries Riemannian surfaces with maximal symmetry groups. The orders of the simmetry groups of these manifolds are multiples of the surfaces genus. This research continues the results published in [1-3].
Acknowledgement. *The work was supported by Institutional Research Program 011303 SATGED of Moldova State University.*

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Geometrical Description of the Orbital Types in the Hydrogen Atom

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Considering the quantum atomic model for hydrogen and relying on some concepts of differential geometry, we calculate the curvature of some types of atomic orbitals. Taking into account this concept of intrinsic curvature [1], a classification of atomic orbitals from a geometric point of view is given. We discuss the implications of characterizing atomic quantum states [2] by the geometric curvatures of the guide curves for these atomic orbitals.

Keywords: Hydrogen; atomic quantum physics; differential geometry; superposition of atomic orbitals; NIST

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On n-ary $T-$quasigroups with a prescribed maximum number of distinct parastrophes

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Let (Q, A) be an n-quasigroup and $\sigma \in S_{n+1}$. The operation ${}^{\sigma}A$, defined by the equivalence: $A(x_1, x_2, \ldots, x_n) = x_{n+1} \Leftrightarrow {}^{\sigma}A(x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(n)}) = x_{\sigma(n+1)}$, is called a parastrophe of (Q, A). In this case the set $H = \{\sigma \in S_{n+1} | {}^{\sigma}A = A\}$ is a subgroup of S_{n+1}. Moreover, the maximum number of distinct parastrophes of (Q, A) is equal to $|S_{n+1} : H|$. Binary finite quasigoups with a given number of distinct parastrophes were studied by C.C. Lindner and D. Steadly [1]. In the ternary case this problem was completely solved for $1, 3, 4, 6, 12$ and 24 distinct parastrophes by M. McLeish [2]. The spectrum of finite ternary quasigroups with exactly 2 or exactly 8 distinct parastrophes is only partially known.
4-Ary quasigroups with a prescribed maximum number of distinct parastrophes and their spectrum are studied in the present work. In particular, we prove that there does not exist 4-ary T-quasigroups (Q, A) having exactly: 1. s distinct parastrophes, where $s = 2, 3, 4, 6, 8, 12, 15$; 2. 10 distinct parastrophes where $\{\sigma \in S_5 | {}^{\sigma}A = A\} \cong A_4$; 3. 20 distinct parastrophes where $\{\sigma \in S_5 | {}^{\sigma}A = A\} \cong Z_6$. Characterizations of 4-ary T-quasigroups with exactly 5 or exactly 10 distinct parastrophes are announced in [3,4]. With give necessary and sufficient conditions when a 4-ary T-quasigroup has exactly 20 distinct parastrophes and estimations of their spectrum.

Theorem 1. *4-ary T-quasigroup (Q, A) with the T-group $(Q, +)$, has exactly 20 distinct parastrophes if and only if there exist $\alpha, \beta \in Aut(Q, +)$ and an element $c \in Q$, such that $\alpha \neq \beta, \alpha \neq I, \beta \neq I$, where $I(x) = -x, \forall x \in Q$, and the operation $A(x_1, x_2, x_3, x_4)$ has one of the following forms: $\alpha x_1 + \alpha x_2 + \alpha x_3 + \beta x_4 + c$, $\alpha x_1 + \alpha x_2 + \beta x_3 + \alpha x_4 + c$, $\alpha x_1 + \beta x_2 + \alpha x_3 + \alpha x_4 + c$, $\beta x_1 + \alpha x_2 + \alpha x_3 + \alpha x_4 + c$, $Ix_1 + Ix_2 + \alpha x_3 + \beta x_4 + c$, $Ix_1 + \alpha x_2 + Ix_3 + \beta x_4 + c$, $Ix_1 + \alpha x_2 + \beta x_3 + Ix_4 + c$, $\alpha x_1 + Ix_2 + Ix_3 + \beta x_4 + c$, $\alpha x_1 + Ix_2 + \beta x_3 + Ix_4 + c$, $\alpha x_1 + \beta x_2 + Ix_3 + Ix_4 + c$.*

Corollary 1. *There exist linear 4-ary T-quasigroups with exactly 20 distinct parastrophes of every order $q \geq 5, q \neq 6$.*

Acknowledgement. *The Institutional Research Program of the Moldova State University for* $2024 - 2027$ *years, subprogram* 011303 *"SATGET", has supported part of the research for this work.*

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On Some Results from Classical Geometry Treated in Indefinite Metric

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In this paper we present some interesting problems and theorems from classical geometry in hyperbolic context. For these results we give interpretations from relativistic physics.

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On the distribution of $\alpha p^2 + \beta$ modulo one and Square-free numbers

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Let α be an irrational number, β a real number, and $a_1, \ldots, a_s$ a set of distinct positive integers that do not form a reduced residue system modulo p^2 for any prime p. In this work, we establish that there are infinitely many prime numbers p that satisfy the condition $||\alpha p^2 + \beta|| < p^{-129/1350}$ and ensure that all the numbers $p + a_1, \ldots, p + a_s$ are square-free.

Keywords: Distribution modulo one, square-free numbers, estimates of exponential sums.
2010 MSC : Primary 11P32, 11J71. Secondary 11L03.

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On the Solutions of Diophantic Equations $7^x + 15^y = z^2$ and $13^x + 12^y = z^2$

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The field of Diophantine equations is ancient, vast, and no general method exists to decide whether a given Diophantine equation has solutions, or how many solutions. In most cases, we are reduced to study individual equations, rather than classes of or many solutions. In the theory of Diophantine equations, is well known the equation $a^x + b^y = z^2$. The literature contains a very large number of articles on such equations [1,2,3,4,5].
In this paper, we solution the equations $7^x + 15^y = z^2$ and $13^x + 12^y = z^2$, where x, y, z are non - negative integer numbers.

Theorem 1. *The Diophantine equation* $7^x + 15^y = z^2$ *has exactly three non-negative integer solutions* $(x, y, z) \in \{(0, 1, 4), (2, 1, 8), (4, 3, 76)\}$.

Theorem 2. *The Diophantine equation* $13^x + 12^y = z^2$ *has exactly three non-negative integer solutions* $(x, y, z) \in \{(1, 1, 5), (2, 5, 499), (3, 1, 47)\}$.

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About some commutative operations with classes of subcategories

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Two operations on the class of subcategories of some categories are defined and conditions are indicated when these operations are commutative.

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Subcategories $\mathcal{B}$-semireflexive

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This paper examines semireflexive subcategories with respect to a class of bimorphisms. The paper investigates the $\mathcal{B}$-semireflexive subcategories in the category $\mathcal{C}_2\mathcal{V}$ of locally convex topological vector spaces, with respect to a given class of bimorphisms [1,2]. The study begins with the known characterizations of semireflexive spaces and extends them in the categorical context by introducing the notions of semireflexive product and its dual. Reflective and coreflective subcategories are

examined through the lens of morphism classes $\varepsilon\mathcal{R}$ and $\mu\mathcal{K}$, establishing a bijective correspondence between certain reflective and coreflective structures. Several theorems are presented that characterize the situations when a subcategory is $(\varepsilon\mathcal{L})$-semireflexive, along with examples including locally complete spaces, inductively semireflexive spaces, $\mathcal{B}$-inductively semireflexive spaces, and p-semireflexive spaces. The results provide categorical descriptions and structural properties of these subcategories, highlighting their interrelations and the role of semireflexive products in their classification.

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7. Computer Science

Evaluating the Performance of SVD and KNN Algorithms in Recommender Systems

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Recommender systems (RS) are an important application of machine learning (ML) that aims to predict user preferences and offer personalized content. This paper proposes a comparative study between Singular Value Decomposition (SVD) and K-Nearest Neighbors (KNN) algorithms based on RS, exploring how each model (matrix factorization approach vs neighborhood-based method) processes user-item interaction, presenting the mathematical tools necessary for constructing their structure, analyzing their performance and evaluating their accuracy metrics. The SVD technique is based on latent factor analysis to capture hidden patterns in item-user interaction, while KNN relies on similarity metrics to recommend items based on user or item neighborhoods. The main goal of this research is to investigate how SVD and KNN can be optimized to get the best recommendation results, based on metrics such as RMSE, MAE, precision and recall.

Keywords: recommender systems, singular value decomposition, factorization, KNN algorithm, networking

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New Mathematical Frontiers in Post-Quantum Cryptography: Research Challenges and Applications

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Post-quantum cryptography (PQC) seeks cryptographic primitives secure against adversaries equipped with large-scale quantum computers. This talk outlines the mathematical foundations of major PQC families (lattice-based, code-based, multivariate, and isogeny-based schemes) and the algebraic, geometric and number-theoretic structures underlying their conjectured hardness [1, 5]. We highlight recent advances in the analysis of lattice reduction, decoding in structured codes and algebraic system solving, together with their implications for the security margins of standardized and emerging protocols [4, 2]. Open problems of particular interest include the complexity of ideal-lattice problems in quantum models, the search for subexponential attacks on multivariate systems and the design of novel hard problems inspired by isogeny graphs [3]. The presentation bridges theoretical mathematics and applied cryptographic engineering, aiming to foster collaboration between pure and applied mathematicians in addressing the urgent challenge of securing information in the quantum era.

Keywords: post-quantum cryptography, lattice problems, code-based cryptography, multivariate systems, isogeny graphs, algorithmic complexity, quantum security.

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Multimodal Understanding of Bessarabian Idioms: Insights from AI-Powered Image generator and Language Models

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This paper presents an innovative approach to preserving and revitalizing Bessarabian idiomatic expressions through the application of Artificial Intelligence (AI) technologies. Recognizing the cultural and linguistic value of these expressions, many of which risk fading into obscurity, we aim to digitally transform a curated collection of 100 idioms by generating vivid representations of both their literal and figurative meanings. Additionally, we analyze the performance of large language models (LLMs) in interpreting these idioms, comparing their ability to capture literal and figurative meanings to ensure accurate multilingual understanding. For this purpose, our team [1] researched the comparative evaluation of five leading AI-powered image generation platforms—Google **Gemini**, **ChatGPT Plus (DALL·E)**, **NightCafe**, **CGDream**, and **OpenArt.ai**—focusing on image quality, generation speed, pricing models, and prompt interpretation accuracy. The analysis reveals that Google Gemini excels in speed and accessibility, while ChatGPT Plus offers high-quality images with strong prompt understanding. NightCafe provides a fast and flexible user experience through its hybrid credit system. However, CGDream struggles significantly with prompt relevance, often producing unrelated images. OpenArt.ai is known for its stylistic versatility and customization features, although it has minor limitations in visual consistency and file handling. The study underscores the importance of aligning the selection of the platform with the specific needs and requirements of the user to ensure optimal results. Using generative AI models, such as ChatGPT Plus, (see Figure 1), we produce vivid visual representations for each expression, while DeepSeek and ChatGPT LLMs aid in multilingual interpretation and context analysis. The figure presents three examples of Bessarabian idiomatic expressions interpreted by ChatGPT and DeepSeek LLM. Both models correctly identified the **literal meanings** of the expressions. However, when it comes to the **figurative meanings**, ChatGPT provided accurate interpretations of all three idioms, while DeepSeek LLM was only correct in one case (the third expression).

This highlights a notable difference in the models' ability to handle culturally nuanced language, with ChatGPT demonstrating a superior understanding of the idiomatic context. As future work, we plan to integrate the collected and annotated idiomatic expressions into a structured *Wikibase* knowledge graph. In addition, these linguistic assets will be incorporated into an interactive AR-based mobile application aimed at educational and cultural engagement. Through immersive visualizations and voice-driven interactions, users will be able to explore idioms in both literal and figurative forms, promoting deeper linguistic comprehension and appreciation of regional heritage. This initiative not only supports the digital preservation of intangible cultural heritage but also showcases the potential of combining AI and AR technologies in modern language learning and cultural storytelling.

Acknowledgement. *This research has been partially supported by COST Action UniDive, CA21167 'Universality, Diversity and Idiosyncrasy in Language Technology', as well as by the project* 25.80012.5007.79SE 'Expressing Cultural Identity through the Digital Transformation of Native Idiomatic Expressions Using AR and AI Technologies, with Implementation in Education'.

Figure 1. Examples of Bessarabian idiomatic expressions interpreted by ChatGPT and DeepSeek LLM

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8. Education

Theoretical foundations of applied mathematics in drone communications

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This paper examines the theoretical mathematical foundations underpinning modern drone communication systems. It discusses key mathematical models that describe signal propagation, interference, and attenuation in dynamic aerial environments. Linear algebra is applied to matrix-based channel modeling and multi-signal processing in space-time domains. Probability theory and stochastic processes are explored as essential tools for modeling noise and random variables affecting data transmission. Fourier and Laplace transforms are presented as fundamental instruments in spectral analysis, facilitating the decomposition of signals and effective noise mitigation. The study also incorporates core concepts from Shannon's information theory, including entropy, source coding, and channel coding, which contribute to enhancing communication efficiency and reliability. The article underscores the integral relationship between mathematical theory and the performance of drone-based communication systems, providing a comprehensive and applicable framework for further research and development in the field.

Keywords: drone communications, linear algebra, stochastic processes, transforms, information theory, signal analysis.

Theoretical foundations of plane geometry with applications in the metric analysis of convex patrulateries

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The paper aims to rigorously substantiate the essential concepts of plane geometry, providing a coherent theoretical framework for the study of geometric figures, with an emphasis on convex quadrilaterals. In the first part, the basic notions of plane geometry are presented – points, lines, angles, distances – as well as the classification and fundamental properties of triangles and quadrilaterals. The emphasis is on the precise definition of convex figures and their characteristic metric relations. In the second part of the paper, convex quadrilaterals are analyzed in detail from a metric perspective, using geometric, trigonometric and analytical methods. The importance of these figures in the construction and demonstration of fundamental theorems of geometry, as well as in educational and technical-scientific applications, is highlighted. The results obtained confirm that the study of convex quadrilaterals, in the context of plane geometry, contributes to

the development of a deep understanding of spatial relations and metric properties, constituting a solid basis for advanced approaches in the field of applied mathematics and geometric education.

Keywords: plane geometry, convex figures, quadrilaterals, metric relations, fundamental theorems, mathematical education.

Enhancing the applied orientation of higher mathematics education in technical university

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At the present stage, the applied orientation of education in higher institutions is becoming increasingly relevant. The article [1] demonstrates that one of the key motivating factors for studying higher mathematics is the ability to connect mathematical concepts with future professional activities. The main directions for implementing the applied orientation of mathematical training for engineering students in technical universities are outlined, including: - taking into account the specifics of education in a technical university; - demonstrating the application of mathematical knowledge in engineering practice during every class; - illustrating the necessity of mathematical tools in studying general technical and specialized subjects through interdisciplinary connections and applied tasks. A comprehensive classification of mathematical problems has been proposed, distinguishing several key types: preparatory, reinforcement, training, exploratory, creative, and assessment tasks. Each type plays a distinct role in the learning process and contributes to the development of different mathematical competencies among engineering students. The article provides a detailed analysis of the purpose and pedagogical function of each category within the context of teaching mathematics at a technical university. For instance, preparatory tasks help students recall prior knowledge and build readiness for new material; reinforcement and training tasks consolidate understanding through repetition and application; exploratory and creative problems encourage independent thinking, innovation, and problem-solving skills; while assessment tasks are used to evaluate students' mastery of the material. Furthermore, the study clearly outlines the criteria and pedagogical requirements for applied mathematical problems, emphasizing their relevance, clarity, and alignment with professional engineering contexts. However, several barriers to the effective use of applied problems during classroom sessions are identified. These include limitations in available instructional time, a lack of adequately developed problem sets that align with both mathematical content and engineering applications, as well as insufficient methodological support for instructors aiming to implement such tasks effectively.

Keywords: higher mathematics, engineering and technical specialties, applied orientation, applied problems

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Digital textbooks for mathematics: between pedagogical potential and implementation challenges

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Over the past decade, the use of digital textbooks in mathematics education has shown a growing trend, influenced by technological advancements, increasing access to digital resources, and the need to adapt education to the demands of the information society. However, the specialized literature reveals both significant benefits and challenges that have limited the impact of these resources on student performance. The objective of this article was to conduct a systematic synthesis of findings from 30 studies published between 2014 and 2024 that investigated the use of digital instructional support materials in mathematics education at primary, secondary, and tertiary levels.

The methodology involved selecting studies based on rigorous criteria of relevance, methodological quality, and geographic coverage, followed by data coding and classification into six major themes: (1) impact on student performance; (2) the teacher's role in mediating digital learning; (3) access and technological infrastructure; (4) quality and interactivity of content; (5) inclusion and student collaboration; (6) sustainability and updating of digital resources. Results were synthesized into a consensus-divergence matrix highlighting areas of agreement as well as controversial aspects.

The analysis revealed broad consensus on the potential of digital textbooks to increase student engagement, stimulate critical thinking, and facilitate personalized learning, yielding positive effects on academic performance in contexts with adequate logistical and pedagogical support. At the same time, significant divergences were identified concern212ing the effectiveness of these tools in the absence of sufficient infrastructure, ongoing teacher training, and adaptation of content to diverse learner needs. It was also observed that digital textbooks with low interactivity tend to be less used and have limited impact, similar to printed versions in digital format.

The conclusions indicate that fully harnessing the pedagogical potential of digital textbooks in mathematics requires an integrated approach combining investment in infrastructure, development of high-quality interactive resources, and continuous professional development for teachers. The study provides an overview of areas of consensus and divergence identified in recent literature, potentially serving as a starting point for policymakers, curriculum designers, and researchers interested in the digitalization of mathematics education.

Acknowledgement. *The study was conducted within the research project* Innovation in the design and exploitation of interactive digital mathematics textbooks for lower secondary education", code: 25.80012.0807.42SE

Keywords: digital textbooks, mathematics, pedagogical potential.

Aerial algebra: concepts, methods and real-world applications

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In recent years, autonomous aerial systems—particularly drones and UAV (Unmanned Aerial Vehicle) formations—have become essential components in various fields such as smart logistics, aerial surveillance, emergency communications, and environmental monitoring. In this context, effective communication and autonomous coordination among aerial entities require the development of rigorous theoretical models capable of capturing the dynamic behavior and spatial distribution of these systems.

This paper proposes an innovative conceptual framework called Aerial Algebra, which aims to provide a mathematical foundation for the interactions between drones within distributed autonomous networks. This framework is built upon abstract algebraic structures, including monoids, groups, and vector spaces, as well as elements from graph theory, topology, formal logic, and information theory. Its primary goal is to construct a formal language that describes and predicts the collective behaviors of drones, with a focus on communication processes, reconfiguration, and distributed decision-making.

The aerial algebra model allows each drone to be represented as an autonomous agent operating within a formal structure defined by a set of compatible operations. These operations include information exchange, synchronization of flight paths, real-time adaptation to obstacles or topological changes, and dynamic redistribution of roles within the network. The resulting structures are dynamic, adaptable, and offer an elegant description of the complexity found in distributed systems.

The applied section of this study includes several case studies, such as:

- autonomous aerial patrol for sensitive perimeters;
- mathematical modeling of drone swarms using linear and parabolic functions, with algebraically generated trajectories;
- variables and loops for path optimization;
- encryption and secured communication via algebraic codes specifically designed for tactical or critical scenarifanasos.

An additional dimension explored is the educational potential of the proposed framework. The study demonstrates the value of integrating aerial algebra into STEAM education, as a pedagogical tool to support the development of modeling, programming, and algorithmic thinking skills. Suggested educational activities, such as simulating flights on mathematically defined trajectories or collaborative drone programming, offer an effective framework for learning applied mathematics in real-world contexts.

The conclusions emphasize that aerial algebra provides a solid foundation for the future development of autonomous aerial communication networks, with applicability in academic research, industry, defense, and education. By combining mathematical formalism with the practical requirements of autonomous aerial control, new interdisciplinary research directions and technological innovation opportunities are opened.

Keywords: Aerial algebra, autonomous systems, aerial communication, drone networks, mathematical modeling, STEAM education.

A comparison between entrance exams to technical faculties in Romania and China

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This article briefly describes the Gaokao exam, which is used for college admissions in China, using some examples from the math test. Next, we compare the Gaokao (syllabus and subjects) with the Romanian entrance exams for math/computer science or technical colleges.

The role of teacher modelling competence in student learning outcomes

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The development of mathematical modelling competence is a subject of study in modern educational research. The analysed studies reflect that the issue of developing mathematical modelling skills needs to be studied both theoretically and practically. A review of two decades of research within the International Community of Teachers of Mathematical Modelling and Applications community, highlighting how teacher education programs can foster modelling competence through teaching-learning labs, technology integration, and pedagogical content knowledge. A technology-supported learning environment can enhance modelling competencies in in-service teachers [1]. It is important to support prospective teachers in developing modelling-specific task competencies through structured teaching labs and reflective practice. In [2] a validated test instrument for assessing modelling competence in teachers is presented. It also discusses international perspectives in shaping global standards for modelling education.

The key challenges in implementing modelling techniques are: conceptual and didactic complexity; curriculum constraints; lack of teacher preparation and confidence; assessment misalignment; student readiness and resistance; resource and time limitations etc.

We can mention some strategies to overcome these challenges: collaborative resource creation - shared repositories of modelling tasks, annotated examples, and visual aids; professional development - focused workshops, modelling labs, and peer mentoring; curriculum design - embedding modelling tasks into units with scaffolding and interdisciplinary links; assessment reform - rubrics that value process, reasoning, and adaptability; student scaffolding - gradual introduction to modelling phases with guided inquiry.

What does "competence in modelling" involve? A definition must cover conceptual understanding; pedagogical strategies; technological fluency; reflective practice. We must highlight that this competence has a social aspect and have impact on students' values and attitudes.

Developing competence in mathematical modelling is not just important for math teachers, it's transformative, because it: bridges theory and reality; fosters critical thinking and creativity; supports interdisciplinary learning; enhances curriculum design and assessment; empowers students for the future.

Acknowledgement. *This work was supported by the National Agency for Research and Development of the Republic of Moldova under project number* 25.80012.0807.66SE Ensuring continuity in the training and development of mathematical modeling competence.

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A problem with 4 non-coplanar points solved in Maple

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In this paper, we propose the use of the Maple environment to solve an analytical geometry problem starting from 4 non-coplanar points in three-dimensional Euclidean space. The classical solution for most of the requirements of the problem involves applying the same formula several times, thus wasting a lot of time. For this reason, we propose an example of automatic solution of the proposed requirements, using the Maple environment.

Keywords: straight line, distance, plane, Maple

Analysis of the disinformation phenomenon in the Republic of Moldova from a social and technological perspective

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The proposed research is driven by the current situation in the Republic of Moldova. Media and Information Literacy (MIL) is a relatively recent field, and the development of competencies in this area is becoming increasingly important, driven by the growing flow of information, which is also on the rise. The pandemic, the war in Ukraine, and parliamentary elections are just a few well-founded reasons for fostering critical thinking and resilience to disinformation among citizens. The low level of media and information literacy poses a threat to democratic principles and represents an increased risk to national security. The article emphasizes the impact of AI in detecting disinformation.

Keywords: disinformation, malinformation, misinformation, fake news, deepfake, social networks, mass media, social media, media and information literacy.

Mathematics – a key foundation for developing programming competence

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The article demonstrates the importance of mathematical competence in the development of digital skills, particularly programming competence. It presents arguments regarding the use of mathematics in writing program code through the application of dynamic programming techniques. The analysis of a competition problem is proposed from the perspective of mathematics and dynamic programming. The article underscores the necessity of integrated STEM education, arguing that programming performance is unattainable without a strong foundation in mathematics.

Keywords: mathematical competence and competence in science, technology, engineering, digital competence, programming competence, programming techniques, dynamic programming, algorithmic complexity.

Alternative Methods for Angle Trisection

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Angle trisection is one of the most well-known classical problems of Euclidean geometry, with origins in ancient Greece, related to the challenge of dividing an arbitrary angle into three equal parts using only a straightedge and compass. Although the impossibility of this construction in general form was rigorously demonstrated in the 19th century, there are alternative methods, based on additional tools and concepts, that allow its realization. This article presents three notable procedures: Archimedes' method, Dürer's method, and Finsler's method, highlighting their geometric and historical principles. The GeoGebra application is used to carry out the step-by-step constructions and to verify the correctness of the proposed solutions. The paper aims to provide an integrated perspective, combining the theoretical approach with interactive applications, in order to highlight both the beauty and the limits of the angle trisection problem.

Valorization on the extension of interactive education in didactic design by implementing interactive worksheets

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The digitalization of education requires the continuous adaptation of teaching strategies and the effective use of modern technological resources in the teaching-learning process. This article analyzes the potential of the "Interactive Education" extension as a support tool in the design and implementation of interactive worksheets, with direct applicability in teaching activities. The main functionalities of the platform are presented, along with the process of creating worksheets and the advantages of using them in educational contexts.

Keywords: Interactive Education, Interactive Worksheets, Digital Instructional Design, Interactive Teaching Activities, Digital Educational Resources.
Acknowledgement. *Study conducted within the scientific research project "Methodology of ICT implementation in the process of studying real sciences from the perspective of the STEAM concept and Artificial Intelligence", code 040101, within the Institutional Research Program (2024-2027), approved by Ministry of Education and Research of the Republic of Moldova, Order No. 102 of 01.02.2024.*

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Comparative Analysis of the Results of the Study of Programming Languages by Future Computer Scientists

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It is known that young computer scientists prefer modern, minimalist high-level programming languages, such as Python, which allow for fast and easy code writing. In the paper, the results

of studying this language by computer science students were analyzed, compared to the results obtained when studying the classic C programming language, the differences between the means of these results were identified, and the statistical significance of the difference in means was identified. It was found that the students' predilection for one programming language or another does not significantly influence the results obtained in its study, but rather depends on many other significant factors, such as algorithmic thinking, individual work skills, continuous practice, etc. Despite studies demonstrating the popularity and demand on the labor market, the students' preference for the Python programming language, however, the results obtained by students of the Computer Science specialty within the FMTI of UPSC do not differ significantly.

Keywords: higher education, curriculum, Computer Science study program, C programming language, Python programming language, statistical analysis.
Acknowledgement. *Study conducted within the scientific research project "Methodology of ICT implementation in the process of studying real sciences from the perspective of the STEAM concept and Artificial Intelligence", code 040101, within the Institutional Research Program (2024-2027), approved by Ministry of Education and Research of the Republic of Moldova, Order No. 102 of 01.02.2024.*

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Digital Pedagogy: Conceepts and Theories

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The multitude of scientific researches in the field of implementing information and communication technologies in the teaching-learning-evaluation process have inevitably contributed to the conceptualization of digital education, in a digital environment, which contributes to the formation of digital competencies, both of teachers and students, and, finally, to the conceptualization of digital pedagogy. Scientists from all over the world have studied the scientific foundations of digital pedagogy, its status within classical pedagogy, learning theories and teaching-evaluation methods in the context of digital pedagogy, contributing by defining this concept from various perspectives and its multidimensional modeling. Thus, the term "digital pedagogy" was first mentioned in the United States of America, Canada and Europe, in the context of the integration of digital technologies in education, and researchers such as: Croxall B., Harris K. D., Beerens K., Harasim L., Keeton L., Fyfe P., Kivunja, Väätäjä J. O., Ruokamo H., Stommel J., Gudova, M.J., Golubinskaia A. V., Demareva V. A., Solovova N. V., Istrate O., Ceobanu C., Cucoș C., Pânișoară I.-O., Gremalschi A. etc. contributed to its conceptualization. The researchers' predilection for digital pedagogy is supported by a series of specialized studies ("Conceptualizing dimensions and a model for digital pedagogy", "Decoding Digital Pedagogy, (Un)Mapping the Terrain", "Digital Pedagogy: Definition and conceptual area", etc.), scientific journals ("Hybrid Pedagogy: the journal of critical digital pedagogy", "Revista de Pedagogie Digitala, ISSN 3008 – 2013", "Digital Pedagogy Cookbook") and dedicated scientific conferences ("A Digital Pedagogy Unconference", "Digital Pedagogy: from didactics to instructional design"), university courses, textbooks and course materials ("Digital Pedagogy"). The paper makes an inroad into the aforementioned studies with the aim of conceptually delimiting "digital pedagogy", identifying the dimensions and modeling their interrelationships.

Keywords: digital pedagogy, digital education, digital skills, digital environment, conceptual model.

Acknowledgement. *Study conducted within the scientific research project "Methodology of ICT implementation in the process of studying real sciences from the perspective of the STEAM concept and Artificial Intelligence", code 040101, within the Institutional Research Program (2024-2027), approved by Ministry of Education and Research of the Republic of Moldova, Order No. 102 of 01.02.2024.*

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Principles of bilingual instruction in mathematics for engineering students

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The subject of this research is the educational process of training engineers at a technical university within a bilingual framework, where professional disciplines are taught in a foreign language (English). The study focuses on both general didactic and specific principles of bilingual instruction in higher mathematics (disciplines of the mathematical cycle). Among the general didactic principles of the modern learner-centered philosophy of education that are relevant to bilingual mathematics instruction, the following are highlighted: the activity-based nature of learning, active interaction among all participants in the educational process, the problem-oriented character of instruction, learner-centered approaches, productivity of learning activities, professional orientation of education, multiculturalism, and humanistic development within the context of cultural dialogue. Taking into account the specifics of teaching mathematics bilingually at a technical university, a set of particular principles is identified: orientation toward achieving the goal of mastering mathematics (as a non-linguistic discipline); the use of both native and foreign languages as

tools of learning; the unity of thinking and speech activity in a foreign language; reliance on both native and foreign languages; rational limitation of communicative sufficiency; interdisciplinary connections and interdependence of different components within the system of engineering training; optimization of the learning process; gradual qualitative complication of educational content; reliance on prior school-level mathematical knowledge; and the development of motivation to acquire the ability and readiness to use a foreign language for professional purposes. The proposed principles of bilingual mathematics instruction at a technical university outline ways of applying the patterns observed in bilingual teaching and learning in accordance with the overarching strategic objective – namely, the development of bilingual subject competence in mathematics among future engineers.

Keywords: foreign language education, bilingual education, model of bilingual teaching/learning, didactic principles, subject-oriented didactic models.

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Teaching Homogenization Theory to Students: A Course and Laboratory Approach

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This paper presents a combined course and laboratory framework for teaching homogenization theory to undergraduate and graduate students. Homogenization is a branch of applied mathematics that studies the effective behavior of heterogeneous materials with rapidly oscillating properties. While the theory is abstract and often considered difficult for students, it can be introduced in an accessible manner through the special case of layered media. We propose a pedagogical strategy integrating rigorous lectures (definitions, lemmas, theorems, proofs) with laboratory activities (worked examples, numerical simulations, and student projects). The case study shows how abstract mathematical concepts can be transformed into a hands-on learning experience.

Effective prompt design strategies: fundamental principles for educators beginning in artificial intelligence

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Communication between a user and a generative model resembles a dialogue between people. The more precisely the interlocutors express their thoughts, the easier it is for them to understand each other. Prompt engineering, however, remains a critical factor that has not been sufficiently explored in this context. This article details the structural components of effective prompts and demonstrates through examples the application of these elements in practical scenarios in an educational context. By integrating prompt engineering and applied linguistics, the research paves the way for the responsible and efficient use of LLMs, transforming theoretical potential into robust practical applications.
Acknowledgement. *Article produced as part of the scientific research project* Methodology of implementing ICT in the process of studying real sciences from the perspective of the STEAM concept and Artificial Intelligence*, project's cod 040101.*

Keywords: Artificial Intelligence, education, prompt, prompt engineering.

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Workshop

Interdisciplinary Perspectives in Digital Pedagogy. Integration of Mobile Educational Applications in Modern Teaching of Technical Topics in Universities

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Workshop description

The workshop proposes a productive session with the participants, on two educational applications developed for Higher Education, which will be presented and analyzed from the perspective of Digital Pedagogy.
The workshop involves four stages, which are connected to the objectives (further presented):

1 An interactive presentation of the concept of Digital Pedagogy and some of its defining characteristics –**O1**;

2 Exemplification of a learning activity carried out with the help of the two applications: 'RM Digital' (Digital Strength of Materials - which includes PDF visualization, audio playback, and interactive animations for asynchronous learning) and 'OM AR' (Machine Parts Augmented Reality - that utilizes augmented reality for exploring 3D models in physical space - **O2**;

3 Generating and supporting an interactive debate - based on a SWOT analysis, performed in working groups - on the interdisciplinary aspects of the learning activity: the psychological dimension of learning, the pedagogical perspective, and the technological perspective when encountering the scientific content, design issues from the gender-balanced perspective – **O3**;

4 Application of a feedback questionnaire and the development of conclusions regarding the integration and utilization of new learning technologies in the didactic process - **O4**.

The workshop leverages the principles of digital pedagogy focused on accessibility, interactivity, awareness of genre-based perception differences, contextual and experiential learning. This approach supports the development of technical skills in a flexible and engaging way for students.

Objectives, relevance

O1 Highlighting specific, defining aspects of Digital Pedagogy

O2 Real-time delivery of a learning activity that utilizes Augmented Reality

O3 Structured debate, on working groups, regarding the strengths, weaknesses, opportunities, and threats of the two applications ('RM Digital' and 'OM AR') from the perspective of Digital Pedagogy, impacting the development of technical skills.

O4 Developing conclusions regarding the integration and capitalization of new learning technologies in the teaching process.

Workshop relevance

The relevance of the workshop derives from:

1 The actuality of the field of the Digital Pedagogy, connected to the continuous process of training for the teaching career and with the diversity of perspectives from which it is approached;

2 The necessity of illustrating a good practice as a support for constructive discussions on the topic;

3 The creation of a collaborative and argumentative working framework in which interdisciplinary and complementary perspectives on the topic can assert and correlate.

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