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dedicated to the memory of
Academician Mitrofan M. Choban

Book of Abstracts

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In Memoriam - Academician Mitrofan Choban

(05.01.1942 – 02.02.2021)

President of the Mathematical Society the Republic of Moldova (1999-2021). Vice-President of the Romanian Society of Applied and Industrial Mathematics (ROMAI) (1995-2021). President of Tiraspol State University (2002-2009). Founder of the school of general topology in the Republic of Moldova. Mathematics professor and researcher at Tiraspol State University for over 50 years. His original contributions to Mathematics can hardly be overestimated.

Mitrofan Cioban was born on January 5, 1942 in Copceac, Tighina county, Moldova (Romania) in the family of the farmers Mihail and Tecla Cioban. He was the fourth of the seven children. His parents encouraged Mitrofan to get as good an education as was possible at the time. They sent him to a boarding school in the neighboring village of Volontiri. Then, in 1959, he graduated from the high school of Volontiri village. After graduation, he worked for a year in the local agricultural organization.

At the age of 17 M. Cioban decided to become a mathematician

As he didn't speak Russian at the time, he had to give up his early dream of becoming a ship designer. At the age of 17 he decided to become a mathematician. In 1960 he was enrolled at Tiraspol State Pedagogical Institute (Moldova), at the Faculty of Physics and Mathematics within Tiraspol State Pedagogical Institute (Tiraspol State University), the first higher education institution in Moldova. Within the Faculty of Physics and Mathematics, the young Mitrofan meets great university professors such as: P. Osmatescu, C. Cozlovschi, M. Cozlovschi, Gh. Gleizer, I. Valuta, etc. Soon he joined a seminar in topology led by Professor P. Osmatescu. Thus, without initially realizing, the topology seminar developed the young student's great passion for research in Topology for a lifetime.

The Topology seminar of Pavel Alexandrov

Having studied one year in Tiraspol, at the initiative of Professor Petru Osmatescu, Mitrofan Cioban and two other young people are sent to study at Moscow State University.

Ion Valuță, being at that time the doctoral student of Professor A. Kurosh, was invited by the Academician P.S. Alecsandrov to attend the conversation with M. Cioban. Professor I. Valuță remembers: "M. Cioban couldn't answer the question asked by P.S. Alexandrov."

Figure 1. Mitrofan Choban at the seminar in Topology led by Professor P. Osmatescu

Either he didn't understand the question well enough, or he couldn't figure out how to answer in Russian. P.S. Alexandrov asked the young people to wait for the results as they would be announced very soon. I thought I should go out too, but he told me to stay. The academician tells me: "You have sent three students to university, but we will admit only two for study "I thanked him, but at the same time I dared to tell him that I had a confusion. He asked me what it was. The young man who did not answer anything, in my opinion, has special abilities for scientific research. P.S. Alexandrov's answer struck me: "If you say so, then we will accept everyone to study. We will always be able to expel the weakest in Mathematics. In a short time, the young mathematician demonstrated his creative potential in Mathematics.

Whithin the Moscow State University, M. Cioban started attending the Topology seminar of Academician Pavel Alexandrov. The scientific coordinator during his years of study at the University was Professor A.V. Arhangel 'skii.

Figure 2. Academician P. S. Alexandrov

Whithin the Topology seminar he did not hesitate to express his opinion when discussing scientific reports of venerable specialists. For example, it was believed that one result of the American mathematician Arthur Stone is final and not subject to further development. At the end of April 1964, while discussing A. Stone's results, unexpectedly student M. Cioban said that this was not the case. And, of course, many doubted that he was right. Nevertheless, M. Cioban did not back down, although many looked with a grin at the insolent student. Pavel Sergeevich turned out to be at his best - he invited Mitrofan Cioban to present his opinion on the development of the idea from the works of A. Stone in a week or, of course, to present his apology to everyone for his daring challenge. A week later, surprisingly to everyone, Cioban presented a wonderful scientific report which formed the basis of his famous scientific article on factorial mappings with separable preimages.

Profesor Stoyan Yordanov Nedev, Bulgaria, remembers: "Apparently, this report put M. Cioban on a special position - he was recognized as a highly qualified specialist in the field of Topology. He finished the third year of study with two excellent works which were soon published in the Journal Reports of the Academy of Sciences of the USSR. Despite the fact that he did not know Russian, German and English, he perfectly knew almost all the works written in these languages over the past 70 years."

This was a generalization of one of A.H. Stone's results in the first paper Mitrofan had referred to. He has proved the following:

Theorem. *If $f : X \to Y$ is a quotient mapping of a metrizable space X onto a Tychonoff first-countable separable space Y, and all fibers under f are separable, then Y is metrizable.*

After this case, P.S. Aleksandrov has repeatedly said: if Mitrofan said this, then this is beyond doubt. P. S. Alexandrov recommended the first article of Mitrofan Cioban containing Theorem 1 for publication in Doklady AN SSSR, one of the most prestigious Soviet mathematical journals. It appeared in print in 1966, and it is his first publication. In fact, his first mathematical result had been obtained earlier, in November 1964, but it was published later.

Professor A.V. Arhagel'skii remembers "Mitrofan became a ≪star≫

in the main seminar on P.S. Alexandrov's General Topology. This made the seminar even more attractive to students, more popular with them. Mitrofan was active not only in Mathematics. He was also a sportsman, participated in Romano-Greco wrestling competitions and won the title of the Champion of Moscow University. At the MSU the name Mitrofan was quite rare and therefore everyone called Mitrofan Cioban simply Mitrofan." It is necessary to mention

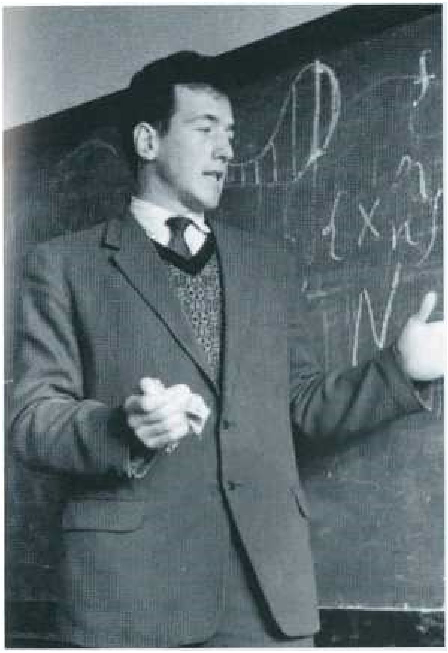

Figure 3. Professor A.V. Arhagel'skii

that student M. Cioban managed to obtain a series of beautiful results in Topology. He published a valuable paper in the prestigious journal Sovietic Mathematiceskij Doklady (see Sov. Math. Dokl). Thus, the publications of M. Cioban during the student period at Moskow University are:

1. On the behavior of metrizability under quotient s-mappings. *Dokl. Akad. Nauk SSSR*, 166:3 (1966), 562–565.
2. Behavior of metrizability under monotone quotient mappings. *Dokl. Akad. Nauk SSSR*, 168:3 (1966), 535–538.
3. Behavior of metrizability under factorial s-mappings. Abstracts. Moscow. *International Congress of Mathematicians.* Section 8, M. (1966), 30.
4. Certain metrization theorems for p-spaces, *Dokl. Akad. Nauk SSSR*, 173:6 (1967), 1270–1272.
5. Finite-to-one open maps, *Dokl. Akad. Nauk SSSR*, 174:1 (1967), 41–44.
6. Perfect mappings and spaces of countable type, *Vestnik Moskov. Univ. Ser. 1. Mat. Mekh.*, 1967, 6, 87–93.

He brilliantly graduated this Faculty in 1967. Here he attracted the attention of the famous and exigent teaching staff. He was recommended to Professor A.V. Arhangel'skii who marked his whole evolution as a mathematician, so that at the beginning of year 1967, he was privileged to study and work in the prestigious collective of researchers in Topology, brilliantly dominated by A.N. Kolmogorov's great personality. A year after his graduation from the faculty he has begun preparing his Doctor's Degree - the chosen specialty being Topology - at the State University "M. Lomonosov" of Moscow. In 1970 he graduated the Ph.D. in Mathematics with the thesis "Relations between classes of topological spaces", his adviser being Professor A.V. Arhangel'skii.

Professional career at Tiraspol State University

Academician Mitrofan Choban was a Mathematics professor and researcher at Tiraspol State University for over 50 years. Professor Mitrofan Choban started his didactic career at Tiraspol State University (Moldova) in 1970 and continuously worked in this university till 2021. He was in succession senior lecturer at the Department of Geometry and Didactics of Mathematics (1970-1974), associate professor at the Department of Geometry and Didactics of Mathematics (1975-1976), head of the Department of Geometry and Didactics of Mathematics (1976-1983), vice-rector of the University for science (1983-2002), president of the University (2002-2009) and head of the Department of Algebra, Geometry and Topology (2009-2021). Since 1981, Professor Mitrofan Choban was adviser for PhD Theses as well as for Dr. Sc. Theses. He advised 22 doctors of sciences and 4 Doctors Habilitatus in Mathematics. His teaching activity concerned the courses of "Geometry" as well as of "Set Theory and Topology". He also taught several special courses: Functional Spaces, Topological Groups, Algebraic Theory of Automata, Topological Universal Algebras, etc.

University of Tbilisi. Presentation of the doctoral thesis in Mathematics

On March 1978, the possibility arose that Mitrofan Choban could present his thesis at the Scientific Council of the University of Tbilisi. In order to carry out the expertise of the thesis, the head of the department, Academician George S. Ciogoshvili, proposed to hold a lecture course on the topic of the thesis and sent the thesis for review to Professor A.V. Arhangel'skii. Mitrofan Choban, assured that all was well and he arrived in Tbilisi in September 1978.

Mitrofan Cioban recalls: "I asked Academician G. S. Ciogoshvili when the meeting of the department will take place. He told me to go to his house the next day at about 12 o'clock. I entered the spacious hall of the apartment. He proposed me to sit in an armchair. After a while he started telling me about the situation of my thesis. Going in front of me, from one corner of the room to the other, he said in Russian: “Now every doctoral dissertation requires a thorough analysis and expertise. A favorable opinion of the thesis is already from Professor A.V. Arhangel’skii. I recently submitted my dissertation to Professor Ponomarev. I personally intend to obtain favorable opinions from all well-known specialists in the field in the country and abroad, even from Michael in the USA. Only after this, the thesis will be examined at the department, as a result, a solid expertise of the thesis will be obtained.” I asked if he would allow me to say something. With the Academician’s permission, I said: “Dear George Severianovici. Thank you for your attention and for the great intentions you have for my modest research. But the following moments appear.

The first moment. Let’s imagine that you have managed to get favorable opinions from all the respective specialists in the world. It does not matter that it will take at least ten years, about half a year for an opinion. In this case, before whom will I defend my thesis? Who will be the official opponent, because everyone approved the thesis positively? Who will be the profile organization? According to your “referendum,” everyone will be “pro.” How will my case be prefected in this case?

The second moment. Yes, why should my work please everyone? Everyone has their own tastes and interests. Let’s say I’m an 18-year-old person who wants to get married. And there is a council of one hundred acsacals (in the East ”acsacal” means an old, respected man with a rich life experience). The condition for me is that I can marry my chosen one, if and only if she is to the liking of all the acsacals. Please tell, ”Will I be married until I’m 60?”

After the last words, I noticed that Academician Ciogoshvili wanted to say something. I was silent waiting. In a few moments he said, “Mitrofan. On my word of honor I no longer send your thesis to anyone. I will also write a positive opinion, and then I will recommend it for support”. He walked over to a closet and pulled out a bottle of vodka. He put it on the table. I opened the bag and took out a bottle of ”Doina” cognac with a box of chocolates. I did a lot that day.

After lunch, everyone in the college already knew the content of our discussion. In the Dean’s office there was Levan Jijiaşvili, chairman of the scientific council, Mirab, secretary of the council, dean of the faculty, a good friend of mine, Lazo Zambahidze. They asked me, ”How did you dare to say that?”. They knew that everything that Academician Ciogoshvili told them was not questioned, but executed. They had feelings of wonder and envy.

Mitrofan Choban remembers ”Towards the New Year 1979 I received a telegram with the content: “Dear Mitrofan. Sincere good wishes, good health and many achievements. Your thesis with three excellent opinions was recommended for defense. Ciogoshvili ”. Life went on. Luckily for me, at the decisive moment, I didn’t know all the Georgian customs.”

In 1980, he became Doctor Habilitatus in Mathematics with the thesis ”Set-valued mappings and their applications” (scientific consultant again being A.V. Arhangel’skii).

Чобан, М.М. (Митрофан Михайлович). **Многозначные отображения и их приложения** : Автореф. дис. на соиск. учен. степ. д-ра физ.-мат. наук : (01.01.04) / М.М. Чобан ; Тбил. гос. ун-т. - Тбилиси, 1979. - 35 с.. - Список работ авт.: с. 34-35 (18 назв.)[MFN: 50324]. UDC: 51.

Figure 4. From the archives of the University of Tbilisi

Scientific performances

In 1995, he was elected corresponding member of the Academy of Sciences of Moldova. Then, in 2000 he was elected Member of the Academy of Moldova, the highest forum of Moldavian spirituality and the highest recognition which a scholar may receive.

His scientific concerns group the following main directions: topology, topological algebra, descriptive set theory, functional analysis, topological optimization theory, measure theory, etc. He solved a number of well-known problems, formulated in the last 100 years by P.S. Alexandrov, A. V. Arhagel'skii, WW Comfort, F. Hausdorff, A. N. Kolmogorov, AI Maltsev, E. Michael, I. Namioka, B.A. Pasynkov, A. Pelczynski, R. Pheps, Z.Frolik, A. Stone, Z. Semadeni etc.

Mitrofan Choban published in academic journals from 1966 to 2021, mostly under the names of Choban or Čoban, and occasionally Ciobanu, Ciobanu, or Coban.

Thus, Professor Choban authored more than 300 papers and 20 books in many branches of Mathematics. He brought important contributions in: Hausdorff's problem on Borelian classes of sets; Alexandroff's problem about the structure of compact subsets of countable pseudocharacter in topological groups; Arhangel'skii's problem on the zero-dimensional representation of topological universal algebras; two Maltsev's problems on free topological universal algebras; two Michael's problems about G -sections of open mappings of compact spaces and of the k-coverings of open compact mappings of paracompact spaces; Phelps' problem about the structure of the set of points of Gateaux differentiability of convex functionals (with P.Kenderov and J.Revalski); Tichonoff's problem about well-possedness of optimization problems in the Banach spaces of continuous functions (with P.Kenderov and J.Revalski); Confort's problem about Baire isomorphism of compact groups; Pasynkov's problem about Raikov completion of topological groups; Arhangel'skii's problem on metrizability of o-metrizable topological groups (with S.Nedev); Pelczinski's and Semadeni's problems about structure of Banach spaces of continuous functions on special compact subsets of quotient spaces of topological groups, etc.

He attended more than 100 scientific forums: 1) International mathematical congresses (Moscow, Zurich, Berlin), conferences (Moscow, New York, Baku, Sofia, Pitesti, Oradea, Sozopol, Bucuresti, Timisoara, Brasov, Chisinau, Novosibirsk, Tbilisi, Lecce, Iasi, Constanta, Sicilia, Livov, Varna, Borovets, Ohrid), 2) symposiums (Prague, Eger, Burgas, Genova, Marseille), 3) All-Union mathematical conferences and symposiums (Minsk, Moscow, Tiraspol, Chisinau, Livov, Sankt-Petersburg, Novosibirsk, Tobolsk, Tartu), 4) several national conferences. Having a great prestige in the world of Mathematics, Professor Mitrofan Choban has been invited to lecture by the well-known institutions: Institute of Mathematics and Informatics of the Academy of Science of Bulgaria, the Universities of Oradea, Tartu, Tbilisi, Tashkent, Tsukuba, Bishkek, North Bay (Canada). Moreover, he was invited as a speaker of the forums: V-th Prague Topol. Symp. (1981), Topological Colloq., Eger, Ungary (1983), International Moscow Topological Conference (1979), Soviet-Japan Topological Symposium, Niigata (1991), Workshop on General Topology and Geometric Topology, Tsukuba (1991), Workshop "Well-Posedness in Optimization, Margarita di Liguri, Italy (1991), International Conference on group theory, Timisoara, Romania (1991), Workshop "Well-Posedness in Stability and Optimization, Sozopol, Bulgaria (1993), Conferences on Applied and Industrial Mathematics, Romania (1994-2019), International Congress of Mathematical Society of South Europe, Borovets, Bulgaria (2003), International Conference "Geometric Topology, Discrete Geometry and Set Theory" in celebration of the centennial of Ljudmila V.Keldysh, Moscow (2004), International Conference "Quality in Formal and non Formal Education", Iasi, Romania (2010), Centennial Conference "Alexandru Myller" Mathematical Seminar", Iasi, Romania (2010), ICTA Islamabad, Pakistan (2011), CAIM - Conferences in Applied and Industrial Mathematics (1993-2019), 8-th International Conference on Applied Mathematics, Baia Mare, Romania (2011), etc..

Due to his prestige in the world of Mathematics he became: 1. Member of the Editorial Boards of: - Buletinul Academiei de Stiinte a Moldovei, Matematica, ROMAI Journal, Scientific Annals of Oradea University, Qusigroups and related systems; 2. President of the Mathematical Society of the Republic of Moldova (1999-2021); 3. Vice-President of the Romanian Society of Applied and Industrial Mathematics (ROMAI) (1995-2021); 4. Member of the Moscow Mathematical Society; 5. Member of the Romanian Mathematical Society.

The special appreciation of his scientific work brought him several prizes, titles and orders, namely: prize of the All-Union Presidium of the ScientificTechnical Societies (1968); prize "Boris Glavan" of the Komsomol of Moldova, in Mathematics (1974); title Excellent of the High Education of the USSR (1980); order "Gloria Muncii" (Glory of Labor) of the Republic of Moldova (2000); State Prize of the Republic of Moldova (2002); Honorary citizen of the Stefan Voda county, Republic of Moldova (2005); prize "Academician Constantin Sibirschi" (2006); Doctor Honorius Causa of the Oradea University (2006); order "Honor" of the Republic of Moldova (2010); Medal "Dmitrie Cantemir" (2007), Medal "Nicolae Milescu Spataru" (2012); 70 years since the creation of the first Research Institutions and 55 of the ASM (2016); Researcher of the Year Award (2016); Order of the Republic of Moldova (2020).

Appreciations and recognitions from the academic world

- "For me there is no doubt that Professor Mitrofan M. Choban

is a world-class scholar."
Professor P. Kenderov, Members of Bulgarian Academy of Sciences

- "M.M. Choban is the most talented mathematician, with a great creative force."
Professor A. Arhangel'skii, Moscow State University

- "The results of M.M. Choban gave rise to a whole series of publications in many countries ..."
Academician A. Fomenko, Russia

- "Many experts in the field of Topology consider it an honor to carry out scientific research together with M. M. Choban"
Professor O. Lupanov and Professor
V. Fedorchuk, Moscow State University

- "Professor M.M. Choban is one of the most famous and recognized topologists in the world. Well known are his substantive research on the theory of multivalued mappings, topological algebras, descriptive theory of sets and function spaces, as well as their numerous applications to other areas of Mathematics."
M. Abel, Professor of the Tartu University, President of the Estonian Mathematical Society.

- "Mitrofan Choban is in the possession of incredible knowledge of the topological phenomena and strong and sophisticated techniques."
Professor G. Skordev, Corresponding Member of the Bulgarian Academy of Sciences

- "Academician Mitrofan Ciobanu is part of the elite of Moldovan scientists. His mathematical, educational and civic work is overwhelming in the field of Mathematics and has addressed new and difficult problems in Topology, Modern Algebra and its Applications."
Academician Radu Miron, "Alexandru Ioan Cuza" University of Iasi.

- "Academician Mitrofan Ciobanu was and remains a star in the world of mathematicians."
Professor Larențiu Calmuțchi, Tiraspol State University

- "It is natural to ask: how does academician Mitrofan Cioban conceive Mathematics? Of course, he sees it in all its complexity, with one small exception - in no way does he perceive it as a form of snobbery. Who but him has tried all the facets: research, teaching, leadership. And every time he succeeded brilliantly, obtaining valuable results, being a talented professor and loved by students, leading Tiraspol State University and the Mathematical Society."
Corresponding member of the Moldovan Academy of Sciences, Professor C. Gaindric;
Corresponding member of the Moldovan Academy of Sciences, Professor S. Cojocaru.

- "I have very many memories of his reign and I am very grateful to him for everything he has done for our countries, for ROMAI, for CAIMs and for me."
Professor Adelina Georgescu, Romania.

- Who did Mitrofan Choban learn from?
M. Choban: „Alexandr Arhangel'skii served me at that time

as a model of professionalism and exemplary conduct ... Many other teachers ... contributed to my training as a specialist. For example, I learned from Vladimir Andrunachievich and Pavel Alexandrov the management of the organization of scientific research, from Otto Schmidt, Andrei Kolmogorov and Andrei Tikhonov - the organization of mathematical applications in various fields, from Anatol Mal'tsev and Alexandr Curosh - universal methods of examining things in depth and at the same time, simple and clear etc. "

- "The founding of the school of General Topology belonged to Mitrofan Choban - a well-known figure in the mathematical sciences. His path to the high peaks of science has not been easy at all, but he has traveled through it well. "
Academician Petru Soltan, Moldova

Figure 5. Academician P. Soltan: "Maestre, ce mai este nou în topologie?"

Professor M. Cioban was a very dear Teacher, respected by students and teachers as well, with a distinguished moral and scientific attitude, a leader with an amazing ability to solve everyone's problems. He was a star that illuminated the students' path and warmed our hearts and all those who knew him. Academician M. Cioban was an exceptional mathematician, but above all, he was an Extraordinary Man. His scientific work is an essential part of Moldova's contribution to Mathematics worldwide.

Liubomir Chiriac, prof., dr. habilitat
Dumitru Cozma, prof., dr. habilitat

Plenary talks

Iterative algorithms for approximating fixed points of enriched contractive mappings in convex metric spaces

Vasile Berinde[1]

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North University Centre at Baia Mare, Baia Mare, Romania
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In a series of very recent papers, the author [2], [1], [3] and his collaborator [4], [5], [6], have used the technique of enrichment of contractive type mappings by Krasnoselskij averaging, introduced in [2], to extend and study some well known classes of mappings. Thus there were introduced and studied the following classes of enriched mappings in Banach spaces: *enriched nonexpansive mappings*, in Hilbert spaces [2]; *enriched contractions* [4], the *enriched Kannan mappings* [5] and the *enriched Chatterjea mappings* [7] etc.

The aim of this paper is to present the class of *enriched almost contractions* which includes most of the above mentioned results as particular cases [8].

In the second part of our presentation, we also survey some contributions of Acad. M. Choban to the Fixed Point Theory.

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On global issues in the study of planar vector fields

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Some of the most significant problems on planar polynomial vector fields are of a global nature. In recent years global studies of families of polynomial systems involving mixed methods, algebraic, geometric as well as analytic were obtained. They provided us with new data acquired with the help of global concepts and numerical and symbolic calculations. Analysing this data leads us to observe some new phenomena.

In this lecture I shall survey these developments and highlight rather intriguing aspects that arose from these studies.

Existence, Uniqueness Solution and Invariant Measure Results for Neutral FSDES in Hilbert Spaces with Non-Lipschitz Coefficients. Controllability

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We study the large time behaviour of the solutions of neutral type stochastic functional-differential equations of the form

$$d[u(t)+g(u_t)] = [Au + f(u_t)]dt + \sigma(u_t)dW(t) \text{ for } t>0; \qquad (1)$$
$$u(t) = \varphi(t), t\in[-h,0),\ \ h>0.$$

Here A is an inifinitesimal generator of a strong continuous semigroup $\{S(t), t\geq 0\}$ of bounded linear operators in real separable Hilbert space H. The noise $W(t)$ is a Q-Wiener process on a separable Hilbert space K. For any $h>0$ denote $C_h := C([-h,0],H)$ to be a space of continuous H-valued functions $\varphi : [-h,0]\to H$, equipped with the norm

$$\|\varphi\|_{C_h} := \sup_{t\in[-h,0]} \|\varphi(t)\|_H,$$

where $\|\cdot\|_H$ stands for the norm in H. The functionals f and g map C_h to H, and $\sigma : C_h \to \mathcal{L}_2^0$, where $\mathcal{L}_2^0 = \mathcal{L}(Q^{1/2}K, H)$ is the space of Hilbert-Schmidt operators from $Q^{1/2}K$ to H. In our studies, the maps f and g do not satisfy the Lipschitz condition. Therefore, it is important for applications.

Finally, $\varphi : [-h, 0] \times \Omega \to H$ is the initial condition, where $(\Omega, \mathcal{F}, P)$ is the probability space.

We study the existence and uniqueness of the solution to the initial problem without the Lipschitz condition. Then we establish the Markov and Feller properties in the shift spaces for such equations, and using the compactness approach we establish the existence of invariant measures in the shift spaces for such equations. The obtained abstract results are applied to stochastic partial differential equations of the reaction-diffusion type. The issue of approximate control of such equations is also studied.

The average number of divisors over sparse sequences

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A basic problem in analytic number theory is to study averages of arithmetic functions. We shall be interested mainly in the class of multiplicative functions, which includes the function counting the number of divisors of a positive integer, also known as the divisor function. One of the first tools to investigate the average order of the divisor function was introduced by Dirichlet, and is referred to as the Dirichlet Hyperbola Method. While this method is certainly useful, it is usually insufficient by itself to deal with a finer question, namely when the sum is taken over sparse sequences. To illustrate this phenomenon, we will first survey some classical results concerning the divisor function over polynomial values. Then we will consider a variant of this problem in the context of modular forms, which are periodic complex functions that satisfy many internal symmetries. More precisely, we will determine the order of magnitude for the divisor sum over the Fourier coefficients of a modular form.

Well-posedness of a nonlinear second-order anisotropic reaction-diffusion problem with nonlinear and inhomogeneous dynamic boundary conditions

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The paper is concerned with a qualitative analysis for a nonlinear second-order boundary value problem, endowed with nonlinear and inhomogeneous dynamic boundary conditions, extending the types of bounday conditions already studied. Under certain assumptions on the input data: $f_1(t,x)$, $w(t,x)$ and $u_0(x)$, we prove the well-posedness (the existence, a priori estimates, regularity and uniqueness) of a classical solution in the Sobolev space $W_p^{1,2}(Q)$. This extends previous works concerned with nonlinear dynamic boundary conditions, allowing to the present mathematical model to better approximate the real physical phenomena (the anisotropy effects, phase change in Ω and at the boundary $\partial\Omega$, etc.).

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Images of compact rings

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Professor V. K. Kharcenko asked many years ago the following: Which compact rings are continuous homomorphic images of compact domains? We solved this question for characteristic $p > 0$. Namely, we proved that every compact local ring of prime characteristic is a continuous homomorphic image of a compact domain. We study also the following question: Which rings are abstract homomorphic images of compact rings? We proved that no infinite finitely generated commutative ring is a homomorphic image of a compact ring.

It is known that every finitely generated commutative ring is residually finite. We could not find the source where this result was proved and included in this paper the proof for completeness. It follows from this result that every commutative finitely generated ring admits a totally bounded ring topology. We give a criterion when every totally bounded ring topology on a commutative finitely generated ring is metrizable.

1. Partial Differential Equations

Hyperbolic equations with piecewise-constant argument of generalized type and nonlocal problems for them

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On the domain $\Omega = [0,T] \times [0,\omega]$ we consider a nonlocal problem for system of hyperbolic equations with piecewise-constant argument of generalized type in the following form

$$\frac{\partial^2 u}{\partial t \partial x} = A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u(t,x) + f(t,x)+$$
$$+A_0(t,x)\frac{\partial u(\gamma(t),x)}{\partial x} + B_0(t,x)\frac{\partial u(\gamma(t),x)}{\partial t} + C_0(t,x)u(\gamma(t),x), \tag{1}$$

$$P(x)u(0,x) + S(x)u(T,x) = \varphi(x), \qquad x \in [0,\omega], \tag{2}$$

$$u(t,0) = \psi(t), \qquad t \in [0,T], \tag{3}$$

where $u(t,x) = colon(u_1(t,x), u_2(t,x), ..., u_n(t,x))$ is unknown vector function, the $n\times n$ matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, $A_0(t,x)$, $B_0(t,x)$, $C_0(t,x)$ and n vector function $f(t,x)$ are continuous on Ω; $\gamma(t) = \zeta_j$ if $t \in [\theta_j, \theta_{j+1})$, $j = \overline{0, N-1}$; $\theta_j \le \zeta_j \le \theta_{j+1}$ for all $j = 0, 1, ..., N-1$; $0 = \theta_0 < \theta_1 < ... < \theta_{N-1} < \theta_N = T$, the $n \times n$ matrices $P(x)$, $S(x)$, and n vector functions $\varphi(x)$ and $\psi(t)$ are continuously differentiable on $[0,\omega]$, $[0,T]$, respectively.

Mathematical modeling of real processes often leads to differential equations with piecewise-constant argument of generalized type and these equations are introduced in the works [1]–[4]. Therefore, the questions of solvability of boundary value problems for such equations are of great importance and relevance.

In the present communication we propose a new approach for solving problem (1)–(3) based on Dzhumabaev's parameterization method [5]. Conditions for the existence and uniqueness to problem (1)–(3) are obtained in the terms of initial data [6].

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Determination of some solutions of the 2D stationary Navier-Stokes equations

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The following system of partial differential equations are examined:

$$\begin{cases} \frac{P_x}{\mu} + uu_x + vu_y = \lambda \triangle u + F_x \\ \frac{P_y}{\mu} + uv_x + vv_y = \lambda \triangle v + F_y \\ u_x + v_y = 0 \end{cases} \tag{1}$$

$P = P(x, y);\ u = u(x, y);\ v = v(x, y);\ F = F(x, y);\ u_x = \frac{\partial u}{\partial x};$
$\triangle u = u_{xx} + u_{yy}; \quad x, y \in \mathbb{R}.$

The system (1) describes the process of plane stationary flow of a liquid or gas. This system represents the Navier-Stokes equations in the case of 2D stationary motion of a viscous incompressible fluid. The P function represent the pressure of the liquid, and u, v functions represent the flow of the liquid or gas, F represents the external forces. The constants $\lambda > 0$ and $\mu > 0$ is a determined parameter of the studied liquid's (of the gas) viscosity and density. We mention here that $a = \dfrac{c}{R_e}$, $c > 0$, where R_e is the Reynolds number.

Applying the method of separation of variables, a series of solutions is determined of system (1).

Doubly Stochastic Yule Cascades and Incompressible Navier-Stokes Equation

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In 1997 Le Jan and Sznitman, [1], have identified a branching stochastic cascade structure behind *deterministic* incompressible Navier-Stokes (NSE) equations in three dimensions. The intuitive idea behind this approach is that the solutions to certain semi-linear evolution PDE, when considered in mild-type formulation in Fourier settings, can be viewed as expectations of a "solution" stochastic functional built on a branching structure – "stochastic cascade". Due to the nature of the NSE non-linearity, the resulting cascades have a doubly stochastic nature, where both exponential branching times, and their intensities are random.

We show that in the case of NSE in scaling-critical settings, the naturally associated stochastic cascade is exploding (i.e. generating infinite number of branches in finite time), which provides an avenue to explore possible lack of well-posedness for associated initial value problems. We illustrate the connection between stochastic explosion and non-uniqueness and finite-time blow up of the solutions in the case of the Montgomery-Smith equation – a simplified model for NSE.

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On a positive semigroup associated to a nonlinear evolution model in an ordered space

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We re-examine the premises of a nonlinear evolution model in an abstract state space, in the sense of Davies, investigated in some recent works. We show that if the abstract state space is a Banach lattice, then the one-parameter C_0 semigroup of positive linear operators, introduced by the assumptions of the model, is necessarily an (abstract) multiplication semigroup. The results are of interest, in particular, in the study of nonlinear kinetic equations, and example of applications to Boltzmann-like equations, Smoluchowski equation, etc, are provided.

On the fundamental solution of the Cauchy problem for equations with negative genus and dissipative parabolicity

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Consider a differential equation with partial derivatives

$$\partial_t u(t;x) = \{A_0(t;i\partial_x) + A_1(t,x;i\partial_x)\}u(t;x), \quad (t;x) \in \Pi_{(0;T]}, \quad (1)$$

where u is an unknown function, $\Pi_Q = \{(t;x) : \, t \in Q, \, x \in \mathbb{R}^n\}$, and

$$A_0(t;i\partial_x) = \sum_{|k|\le p} a_{0,k}(t) i^{|k|}\partial_x^k, \quad A_1(t,x;i\partial_x) = \sum_{|k|\le p_1} a_{1,k}(t;x) i^{|k|}\partial_x^k$$

are differential expressions of orders p and p_1, respectively. We assume that Eq.

$$\partial_t u(t;x) = A_0(t;i\partial_x)u(t;x), \quad (t;x) \in \Pi_{(0;T]}, \quad (2)$$

is parabolic according to Shilov on the set $\Pi_{[0;T]}$ with the parabolicity index h, $0 < h \leq p$, and genus $\mu < 0$, and the order p_1 of the group of junior members of the Eq. (1) is less than h [1]: $0 \leq p_1 < h$.

Using the properties of the fundamental solution of the Cauchy problem for Eq. (3) studied in [2], the correctness of the following statement is proved.

Theorem. *Let the coefficients $a_{0,k}(t)$ and $a_{1,k}(t;x)$ of the Eq. (1) on the set $\Pi_{[0;T]}$ be continuous with respect to the variable t, are infinitely differentiable with respect to the variable x and are bounded together with their derivatives. Then, for Eq. (1), there exists a fundamental solution $Z(t,x;\tau,\xi)$ of the Cauchy problem, which is differentiable with respect to the variable t and infinitely differentiable with respect to each of the variables x and ξ. The following assessments are also correct:*

$$\exists \delta > 0 \; \forall \{r,q\} \subset \mathbb{Z}_+^n \; \exists c > 0 \; \forall \, 0 \leq \tau < t \leq T \; \forall \{x;\xi\} \subset \mathbb{R}^n :$$

$$|\partial_\xi^r \partial_x^q Z(t,x;\tau,\xi)| \leq c(t-\tau)^{-\frac{n+|r+q|}{h}} e^{-\delta \frac{\widetilde{|x-\xi|}^\lambda}{(t-\tau)^\gamma}}$$

(here $\widetilde{|x|}^\lambda := |x_1|^\lambda + \ldots + |x_n|^\lambda$, $\lambda := \frac{1}{1-\mu/h}$ and $\gamma := \frac{1}{h-\mu}$).

This information about the Z function is important for building the classical theory of the Cauchy problem for parabolic equations with negative genus and variable coefficients.

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On the solvability of singular integral equations

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We propose a method for solving singular integral equations perturbed by compact operators. In the monographs of N. Muskhelishvili, F. Gakhov and other works it is indicated that the solution of singular integral equations can be found in rare cases. Even in these cases finding the exact solution requires a complex calculation of the singular integrals, which is accompanied with large theoretical and computational difficulties.

In this talk we deal with the problem of solving singular integral equations containing compact terms. Each operator A defined by the left side of this equation is associated with a matrix operator $\tilde{A}$ that has the property that both operators are or are not invertible in the respective spaces.

Thus, the solvability of the considered equation is reduced to a similar problem for a system of equations, which turns out to be a system of "ordinary" singular integral equations without compact terms. The obtained system of singular integral equations is solved by the coefficient factorization method developed in [1]. An explicit expression is determined for the solution of the original equation in terms of the solution of the system of equations. The method presented in this talk is based on the results of the work [2] and can be used to solve various classes of functional equations with composite operators that fall under the general scheme developed in [2].

To invert operators of the form $\tilde{A} = aP + bQ$, where a and b are matrices of continuous functions with conditions $\det a \neq 0$, $\det b \neq 0$, it is necessary to factorize the matrix $c = b^{-1}a$. This means that the matrix c must be represented in the form

$$c = c_- \cdot diag(t^{\kappa_1}, t^{\kappa_2}, ..., t^{\kappa_n}) \cdot c_+ ,$$

where $c_+\,(c_-)$ are the matrices of the function with analytic elements in the domains $F^+ = \{z|\ |z| < 1\}\,(F^- = \{z|\ |z| > 1\})$, and $\kappa_1, \kappa_2, ..., \kappa_n$ are integers called the partial indices of the operator $\tilde{A}$. Depending on the numbers $\kappa_1, \kappa_2, ..., \kappa_n$, the operator $\tilde{A}$ can be invertible, left or right invertible. In particular, if all numbers $\kappa_1, \kappa_2, ..., \kappa_n$ are positive, then the operator $\tilde{A}$ is left invertible, if all numbers $\kappa_1, \kappa_2, ..., \kappa_n$ are negative, then $\tilde{A}$ is right invertible, and finally, if all numbers are equal to zero, then $\tilde{A}$ it is invertible. We apply these results to the inversion of the operator $\tilde{A}$.

Finding the operator $\tilde{A}$, being given the operator A, is based on the concept of extension of a linear operator.

Theorem. *Each element A from the algebra V of the form*

$$A = \sum_{j=1}^{r} A_{j1}A_{j2}\cdots A_{js}\ \ (A_{jk} \in V)$$

admits linear stretching ($m \leq r(s+1)+1,\ m = rang\tilde{A}$).

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Convergence estimates for solutions to the semilinear plate equation with small parameter

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Let $\Omega \subset \mathbb{R}^n$ be an open and bounded set with the smooth boundary $\partial\Omega$. Consider the following initial boundary value problem for the plate equation, which in what follows will be called (P_ε):

$$\begin{cases} \varepsilon\, u_{tt}(x,t) + u_t(x,t) + \Delta^2 u(x,t) + B(u(t)) = f(x,t), & (x,t) \in \Omega \times (0,T) \\ u\big|_{t=0} = u_0(x), \quad u_t\big|_{t=0} = u_1(x), \quad x \in \Omega \\ u\big|_{x\in\partial\Omega} = \dfrac{\partial u}{\partial \bar{\nu}}\Big|_{x\in\partial\Omega} = 0, \quad t \geq 0, \end{cases} \tag{P_ε}$$

where $\bar{\nu}$ is the outer normal vector to $\partial\Omega$ and ε is a small positive parameter.

We study the behaviour of the solutions to the problem (P_ε) relative to the corresponding solutions to the unperturbed problem:

$$\begin{cases} v_t(x,t) + \Delta^2 v(x,t) + B(v(t)) = f(x,t), & (x,t) \in \Omega \times (0,T) \\ v\big|_{t=0} = u_0(x), \quad x \in \Omega \\ v\big|_{x\in\partial\Omega} = \dfrac{\partial v}{\partial \bar{\nu}}\Big|_{x\in\partial\Omega} = 0, \quad t \geq 0, \end{cases} \tag{P_0}$$

as $\varepsilon \to 0$. We consider the case when the operator B is Lipschitz and the case when the operator B is monotone.

Under some conditions on u_0, u_1 and f we prove that

$$u \to v \quad \text{in} \quad C([0,T]; L^2(\Omega)) \cap L^\infty(0,T; H^2(\Omega)), \quad \text{as} \quad \varepsilon \to 0. \tag{1}$$

This means that in the indicated norms the perturbation (P_ε) of the system (P_0) is regular.

Moreover, we prove that

$$u' - v' - \alpha\, e^{-t/\varepsilon} \to 0 \quad \text{in} \quad C([0,T]; L^2(\Omega)) \cap L^\infty(0,T; H^2(\Omega)) \quad \alpha \neq 0, \tag{2}$$

as $\varepsilon \to 0$. It means that the derivatives of solutions to the problem (P_ε) does not converge to the derivatives of the corresponding solutions to problem (P_0), as $\varepsilon \to 0$. The relation (2) shows that the derivative u' has a singular behaviour in the neighborhood of $t = 0$ as $\varepsilon \to 0$. This singular behaviour is determined by the function $\alpha\, e^{-t/\varepsilon}$, which is *the boundary layer function,* and the neighborhood of $t = 0$ is *the boundary layer* for u'.

The proofs of the relations (1) and (2) are based on two key points. The first one is the relationship between the solutions to the problems (P_0) and (P_ε) in the linear case. The second key point are the *a priori* estimates of the solutions to the problem (P_ε), which are uniform relative to the small parameter ε.

2. Ordinary Differential Equations, Dynamical Systems

Implicit symplectic methods for high precision numerical integration of the Solar System

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We present FCIRK16, a 16th-order implicit symplectic integrator for longterm high-precision Solar System simulations. Our integrator takes advantage of the near-Keplerian motion of the planets around the Sun by alternating Keplerian motions with corrections accounting for the planetary interactions. Compared to other symplectic integrators (the Wisdom and Holman map and its higher-order generalizations) that also take advantage of the hierarchical nature of the motion of the planets around the central star, our methods require solving implicit equations at each time-step. We claim that, despite this disadvantage, FCIRK16 is more efficient than explicit symplectic integrators for high-precision simulations thanks to: (i) its high order of precision, (ii) its easy parallelization, and (iii) its efficient mixed-precision implementation which reduces the effect of roundoff errors. In addition, unlike typical explicit symplectic integrators for near-Keplerian problems, FCIRK16 is able to integrate problems with arbitrary perturbations (non-necessarily split as a sum of integrable parts). We present a novel analysis of the effect of close encounters in the leading term of the local discretization errors of our integrator. Based on that analysis, a mechanism to detect and refine integration steps that involve close encounters is incorporated

in our code. That mechanism allows FCIRK16 to accurately resolve close encounters of arbitrary bodies. We illustrate our treatment of close encounters with the application of FCIRK16 to a point-mass Newtonian 15-body model of the Solar System (with the Sun, the eight planets, Pluto, and five main asteroids) and a 16-body model treating the Moon as a separate body. We also present some numerical comparisons of FCIRK16 with a state-of-the-art high-order explicit symplectic scheme for 16-body model that demonstrate the superiority of our integrator when very high precision is required.

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The Multifrequency Systems with Linearly Transformed Arguments and Multipoint and Local-Integral Conditions

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A problem of existence and uniqueness of the solution and its approximate construction for the system of differential equations with a vector of slow variables $a \in \Delta \subset \mathbb{R}^n$ and fast variables $\varphi \in T^m$ of the form

$$\frac{da}{d\tau} = X(\tau, a_\Lambda, \varphi_\Theta), \quad \frac{d\varphi}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + Y(\tau, a_\Lambda, \varphi_\Theta), \tag{1}$$

are investigated in the paper. Here $\tau \in [0, L]$, small parameter $\varepsilon \in (0, \varepsilon_0]$, $0 < \lambda_1 < \cdots < \lambda_p \leq 1$, $0 < \theta_1 < \cdots < \theta_q \leq 1$, $a_\Lambda = (a_{\lambda_1}, \ldots, a_{\lambda_p})$, $a_{\lambda_i}(\tau) = a(\lambda_i \tau)$, $\varphi_\Theta = (\varphi_{\theta_1}, \ldots, \varphi_{\theta_q})$, $\varphi_{\theta_j}(\tau) = \varphi(\theta_j \tau)$.

Multifrequency ODE systems are researched in detail in [1], systems with delayed argument were studied in [2, 3], etc.

Conditions are set for the system (1)

$$\begin{aligned} \sum_{\nu=1}^{r} \alpha_\nu a(\tau_\nu) &= \sum_{\nu=1}^{s} \int_{\xi_\nu}^{\eta_\nu} f_\nu(\tau, a_\Lambda, \varphi_\Theta) d\tau, \\ \sum_{\nu=1}^{r} \beta_\nu \varphi(\tau_\nu) &= \sum_{\nu=1}^{s} \int_{\xi_\nu}^{\eta_\nu} g_\nu(\tau, a_\Lambda, \varphi_\Theta) d\tau, \end{aligned} \tag{2}$$

where $[\xi_\nu, \eta_\nu] \subset [0, L]$, $\bigcap\limits_{\nu} [\xi_\nu, \eta_\nu] = \emptyset$.

For the problem (1), (2) a much simpler problem is constructed by averaging over fast variables φ_{θ_ν} on the cube of periods $[0, 2\pi]^{mq}$.

A sufficient condition for the system of equations (1) to exit a small circumference of the resonance of frequencies $\omega(\tau)$ was found, the condition of which in the point $\tau \in [0, L]$ is $\sum\limits_{\nu=1}^{q} \theta_\nu \big(k_\nu, \omega(\theta_\nu \tau)\big) = 0, k_\nu \in Z^m, \|k_1\| + \cdots + \|k_q\| \neq 0.$

It is proved that for the smooth enough right parts of the system (1) and sub-integral functions under conditions (2), the condition to exit the circumference of resonances and for small enough $\varepsilon_0 > 0$ there exists a unique solution for the problem (1), (2) and an estimation is found

$$\|a(\tau; y, \psi, \varepsilon) - \overline{a}(\tau; \overline{y})\| + \|\varphi(\tau; y, \psi, \varepsilon) - \overline{\varphi}(\tau; \overline{y}, \overline{\psi}, \varepsilon)\| \leq c_1 \varepsilon^{\alpha}, \alpha = (mq)^{-1},$$

where $c_1 > 0$ and does not depend on ε, $\big(a(0; y, \psi, \varepsilon), \varphi(0; y, \psi, \varepsilon)\big) = (y, \psi)$, $\big(\overline{a}(\tau; \overline{y}), \overline{\varphi}(\tau; \overline{y}, \overline{\psi}, \varepsilon)\big)$ – the solution of the averaged problem with initial conditions $(\overline{y}, \overline{\psi})$, while $\|y - \overline{y}\| + \|\psi - \overline{\psi}\| \leq c_2 \varepsilon^{\alpha}$.

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Geometrical classification of a family of cubic systems

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In this article we consider the class $\mathbf{CSL}_7^{2r2c\infty}$ of non-degenerate real planar polynomial cubic vector fields, which possess two real and two complex distinct infinite singularities and invariant straight lines of total multiplicity 7, including the line at infinity.

This article is a continuation of [1] in which the classification of a subfamily of this kind of systems is done in the case when the invariant affine lines form a configuration of the parallelism type $(3,3)$.

Here we classify the subfamily of cubic systems in $\mathbf{CSL}_7^{2r2c\infty}$, possessing configurations of invariant line of the parallelism type $(3,1,1,1)$, according to the relation of equivalence of configurations [2]. A configuration of invariant lines will be said to the of parallelism type $(3,1,1,1)$ if there exist one triplet and 3 additional lines in four distinct directions. Moreover, each invariant line, including the line at infinity of the system, is endowed with its own multiplicity and together with all the real singular points of this system located on these invariant lines, each one endowed with its own multiplicity. We denote this subfamily by $\mathbf{CSL}_{(3,1,1,1)}^{2r2c\infty}$.

Our resuts are following ones:

1. We prove that there are exactly 42 distinct configurations of the type $(3,1,1,1)$. Moreover we construct all the orbit represen-

tatives of the systems in this class with respect to affine group of transformations and a time rescaling.

2. Necessary and sufficient the affine invariant conditions for the realization of each one the mentioned configurations are constructed, in terms of polynomial invariants [3], [4].

3. Using some geometric invariants we defined we prove that all 42 configurations are realizable within the class $\mathbf{CSL}^{2r2c\infty}_{(3,1,1,1)}$ and are non-equivalent.

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On the family $\boldsymbol{QSL_3}$ of quadratic systems with invariant lines of total multiplicity exactly 3

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Let $\boldsymbol{QSL_{\geq i}}$ be the family of quadratic differential systems with invariant lines of total multiplicity at least i and let $\boldsymbol{QSL_i}$ denote the family of quadratic systems with invariant lines of total multiplicity exactly i. For any polynomial system the line at infinity is invariant. Thus the family $\boldsymbol{QS}$ of *all* quadratic systems is the same as $\boldsymbol{QSL_{\geq 1}}$.

In the papers [1-4] the families of systems QSL_i with $i = 4, 5, 6$ (6 is the maximum number of invariant lines which could have a quadratic system) were completely studied including first integrals and phase portraits. Now we are interested in systems belonging to the family QSL_3. We mention that up to now three subfamilies in QSL_3 have been investigated. More precisely: (i) the class of Lotka-Volterra systems possessing 2 real invariant straight lines intersecting at a finite real point [5]; (ii) the class of quadratic systems possessing 2 complex invariant lines intersecting at a finite real point [6]; (iii) the class of quadratic systems possessing two invariant lines (real or complex) intersecting at an infinite real point [7].

We point out that only for the subfamily (*iii*) there were investigated its limit points within QSL_3. To complete the study of the whole family QSL_3 we consider here the limit points of the subfamilies (*i*) and (*ii*) within QSL_3.

The main result is the following: A quadratic system in QSL_3 possesses one of the 82 possible configurations of invariant lines. Moreover we determine the affine invariant conditions in terms of the invariant polynomials for the realization of each one of these 82 configurations.

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Averaging Principle on Semi-axis for Semi-linear Differential Equations

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We establish an averaging principle on the real semi-axis for semi-linear equation

$$x' = \varepsilon(\mathcal{A}x + f(t) + F(t,x)) \tag{1}$$

with unbounded closed linear operator $\mathcal{A}$ and asymptotically Poisson stable (in particular, asymptotically stationary, asymptotically periodic, asymptotically quasi-periodic, asymptotically almost periodic, asymptotically almost automorphic, asymptotically recurrent) coefficients (see [1] for details). Under some conditions we prove that there exists at least one solution, which possesses the same asymptotically recurrence property as the coefficients, in a small neighborhood of the stationary solution to the averaged equation, and this solution converges to the stationary solution of averaged equation uniformly on the real semi-axis when the small parameter approaches to zero.

Consider the following differential equation

$$x'(t) = \mathcal{A}x(t) + f(t) + F(t, x(t)), \tag{2}$$

where $f \in C(\mathbb{R}_+, \mathfrak{B})$, $F \in C(\mathbb{R}_+ \times \mathfrak{B}, \mathfrak{B})$ and $\mathcal{A} : D(\mathcal{A}) \to \mathfrak{B}$ is a linear operator acting from $D(\mathcal{A}) \subseteq \mathfrak{B}$ to $\mathfrak{B}$.

We will consider a differential equation (2) when the linear operator $\mathcal{A}$ is an infinitesimal operator which generates a C_0-semigroup $\{U(t)\}_{t\geq 0}$.

Definition. *A semigroup of operators $\{U(t)\}_{t\geq 0}$ is said to be hyperbolic if there is a projection $\mathcal{P}$ and constants $\mathcal{N}, \nu > 0$ such that each $U(t)$ commutes with $\mathcal{P}$, $U(t) : Im\mathcal{Q} \mapsto Im\mathcal{Q}$ is invertible and for every $x \in E$*

$$|U(t)\mathcal{P}x| \leq \mathcal{N}e^{-\nu t}|x|, \quad \textit{for } t \geq 0;$$

$$|U_{\mathcal{Q}}(t)x| \leq \mathcal{N}e^{\nu t}|x|, \quad \textit{for } t < 0;$$

where $\mathcal{Q} := I - \mathcal{P}$ and, for $t < 0$, $U_{\mathcal{Q}}(t) := [U(-t)\mathcal{Q}]^{-1}$.

Denote by Ψ the family of all decreasing, positive bounded functions $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ with $\lim\limits_{t\to+\infty} \psi(t) = 0$.

Below we will use the following conditions:

(G1): $F(t, 0) = 0$ for any $t \geq 0$;

(G2): there exists a positive constant L such that

$$|F(t, x_1) - F(t, x_2)| \leq L|x_1 - x_2|$$

for any $x_1, x_2 \in \mathfrak{B}$ and $t \in \mathbb{R}_+$;

(G3): there exists functions $\bar{F} \in C(\mathfrak{B}, \mathfrak{B})$ and $\omega : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ (respectively, an element $\bar{f} \in \mathfrak{B}$ and function $\omega \in \Psi$) such that

$$\frac{1}{T}\Big|\int_t^{t+T} [F(s, x) - \bar{F}(x)]dt\Big| \leq \omega(T, r)$$

(respectively,

$$\frac{1}{T}\Big|\int_t^{t+T} [f(s) - \bar{f}]dt\Big| \leq \omega(T) \quad)$$

and $\omega(\cdot, r) \in \Psi$ for any $t \in \mathbb{R}_+$, $T > 0$, $r > 0$ and $x \in B[0, r]$.

The standard form of (2) is

$$x'(t) = \varepsilon(\mathcal{A}x(t) + f(t) + F(t, x(t))). \tag{3}$$

We will consider also the following equations

$$x'(t) = \mathcal{A}x(t) + f(\frac{t}{\varepsilon}) + F(\frac{t}{\varepsilon}, x(t))), \tag{4}$$

where $f_\varepsilon(t) := f(\frac{t}{\varepsilon})$ (respectively, $F_\varepsilon(t,x) := F(\frac{t}{\varepsilon}, x))$ for any $t \in \mathbb{R}_+$ (respectively, for any $(t,x) \in \mathbb{R}_+ \times \mathfrak{B}$), $\varepsilon \in (0, \varepsilon_0]$ and ε_0 is some fixed small positive number. Along with equations (3)-(4) we will consider the following averaged differential equation

$$x'(t) = \mathcal{A}x(t) + \bar{f} + \bar{F}(x(t)). \tag{5}$$

Theorem. *Suppose that the following conditions hold:*

1. *$-\mathcal{A}$ is a sectorial hyperbolic operator;*
2. *the function F satisfies conditions (**G1**)-(**G3**);*
3. *the functions f and F are Lagrange stable;*
4. $$L < \frac{\nu}{2\mathcal{N}} \ ,$$
   *where $\mathcal{N}$ and ν there are the numbers figuring in the **Definition**.*

Then there exists a positive number ε_0 such that for any $0 < \varepsilon \leq$ ε_0

1. *equation (4) has a unique solution $\psi_\varepsilon \in C_b(\mathbb{R}_+, \mathfrak{B})$ with $\mathcal{P}\psi_\varepsilon(0) = 0$;*

2. *if the function* $f \in C_b(\mathbb{R}_+,\ \mathfrak{B})$ *is asymptotically stationary (respectively,* τ*-periodic, quasi-periodic, Bohr almost periodic, Bohr almost automorphic, Birkhoff recurrent, positively Lagrange stable), then equation (1) has a unique solution* $\varphi_\varepsilon \in C_b(\mathbb{R}_+, \mathfrak{B})$ *with* $\mathcal{P}\varphi_\varepsilon(0) = 0$ *which is asymptotically stationary (respectively,* τ*-periodic, quasi-periodic, Bohr almost periodic, Bohr almost automorphic, Birkhoff recurrent, positively Lagrange stable);*

3.
$$\lim_{\varepsilon\to 0} \sup_{t\in\mathbb{R}_+} |\psi_\varepsilon(t) - \bar{\psi}| = 0,$$

where $\bar{\psi}$ *is a unique stationary solution of equation (5).*

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About stability of linear systems with delay

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This paper investigates the application of approximation schemes for differen-tial-difference equations [1-3] to construct algorithms for the approximate fin-ding of nonsymptotic roots of quasipolynomials and their application to study the stability of solutions of systems of linear differential equations with delay.

Consider the initial problem for a system of differential-difference equations

$$\frac{dx}{dt} = Ax(t) + \sum_{i=1}^{k} B_i x(t - \tau_i), \tag{1}$$

$$x(t) = \varphi(t), t \in [-\tau, 0], \tag{2}$$

where $A, B_i, i = \overline{1,k}$ fixed $n \times n$ matrix, $x \in R^n, 0 < \tau_1 < \tau_2 < ... < \tau_k = \tau$.

Let us correspond to the initial problem (1) - (2) the system of ordinary differential equations [1-2]

$$\begin{aligned} \frac{dz_0(t)}{dt} &= A(t)z_0(t) + \sum_{i=1}^{k} B_i z_{l_i}(t), \qquad l_i = [\frac{\tau_i m}{\tau}], \\ \frac{dz_j(t)}{dt} &= \mu(z_{j-1}(t) - z_j(t)), \quad j = \overline{1,m}, \quad \mu = \frac{m}{\tau}, m \in N, \end{aligned} \tag{3}$$

$$z_j(0) = \varphi(-\frac{\tau j}{m}), \quad j = \overline{0,m}. \tag{4}$$

Theorem 1 [2]. *Solution of the Cauchy problem (3)-(4) approximates the solution of the initial problem (1)-(2) at $t \in [0,T]$ if $m \to \infty$.*

Theorem 2 [1]. *If the zero solution of the system with delay (1) is exponentially stable (not stable), then there is $m_0 > 0$ such that for all $m > m_0$, the zero solution of the approximating system (3) is also exponentially stable (not stable). If for all $m > m_0$ the zero solution of the approximation system (3) is exponentially stable (not stable) then the zero solution of the system with a delay (1) is exponentially stable (not stable).*

It follows from Theorem 2 that the asymptotic stability of the solutions of the delayed linear equations approximating system of ordinary differential equations for sufficiently large values of m are equivalent. This fact will be used to study the stability of linear differential-difference equations [3].

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Solving boundary value problems for linear neutral delay differential-difference equations using a spline collocation method

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In this work, an iterative scheme using cubic splines with defect two is considered for a boundary value problem for neutral delay linear differential-difference equations. The conditions for the boundary value problem solution existence for various classes of differential-difference equations were considered in [1-3].

Let us consider the following boundary value problem

$$y''(x) = \sum_{i=0}^{n} \Bigg(a_i(x)\, y(x - \tau_i(x)) + b_i(x)\, y'(x - \tau_i(x)) + \tag{1}$$

$$+ c_i(x)\, y''(x - \tau_i(x)) \Bigg) + f((x),$$

$$y^{(p)}(x) = \varphi^{(p)}(x),\ p = 0, 1, 2,\ x \in [a^*; a],\ y(b) = \gamma, \tag{2}$$

where $\tau_0(x) = 0$ and $\tau_i(x),\ i = \overline{1, n}$ are continuous nonnegative functions defined on $[a, b]$, $\varphi(x)$ is a continuously differentiable function given on $[a^*; a]$, $\gamma \in R$,

$$a^* = \min_{0 \le i < n} \left\{ \inf_{x \in [a; b]} (x - \tau_i(x)) \right\}.$$

We introduce the sets of points determined by the delays $\tau_1(x), \ldots, \tau_n(x)$:

$$E_{i1} = \Big\{ x_j \in [a, b] :\ x_j - \tau_i(x_j) = a,\ j = 1, 2, \ldots \Big\},$$

$$E_{i2} = \Big\{ x_j \in [a, b] :\ x_0 = a,\ x_{j+1} - \tau_i(x_{j+1}) = x_j,\ j = 0, 1, 2, \ldots \Big\},$$

$$E_2 = \bigcup_{i=1}^{n} (E_{i1} \cup E_{i2}).$$

A function $y = y(x)$ is called a solution of the problem (1)-(2) if it satisfies the equation (1) on $[a; b]$ (with the possible exception of the set E_2) and boundary conditions (2).

We will set an irregular grid $\Delta = \{a = x_0 < x_1 < \ldots < x_m = b\}$ on $[a; b]$ such that $E_2 \subset \Delta$.

For finding an approximate solution of the boundary value problem (1)-(2) a computational scheme in the form of a sequence of cubic splines with defect 2 on the grid Δ is proposed and substantiated [4].

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Center conditions for a cubic differential system having an integrating factor

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We consider the cubic system of differential equations

$$\dot{x} = y + p_2(x,y) + p_3(x,y),\ \dot{y} = -x + q_2(x,y) + q_3(x,y), \qquad (1)$$

where $p_j(x, y)$, $q_j(x, y) \in \mathbb{R}[x, y]$ are homogeneous polinomials of degree j. The origin $O(0, 0)$ is a singular point for (1) with purely imaginary eigenvalues, i.e. a focus or a center. The problem of distinguishing between a center and a focus (the problem of the center) is open for general cubic systems.

In [1] the problem of the center was solved for cubic system (1) with: four invariant straight lines; three invariant straight lines; two invariant straight lines and one irreducible invariant conic. The center conditions for a cubic differential system (1) with two invariant straight lines and one irreducible invariant cubic curve $\Phi \equiv a_{30}x^3 + a_{21}x^2y + a_{12}xy^2 + a_{03}y^3 + x^2 + y^2 = 0$ were found in [2] and for cubic system (1) having an integrating factor $\mu^{-1} = \Phi^h$ were found in [3], where $a_{30}, a_{21}, a_{12}, a_{03}$ and h are real parameters.

In this talk we give the conditions under which the cubic system (1) has an integrating factor of the form

$$\mu^{-1} = \Psi^h, \tag{1}$$

where $\Psi \equiv a_{20}x^2 + a_{11}xy + a_{02}y^2 + a_{10}x + a_{01}y + 1 = 0$ is an irreducible invariant conic and $a_{20}, a_{11}, a_{02}, a_{10}, a_{01}$ and h are real parameters.

According to [2] the cubic differential systems (1) which have integrating factors of the form (1) have a center at the singular point $O(0, 0)$.

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Invariants of E. Cartan and their applications to the theory of differential equations

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In the presented report, the theory of E. Cartan's invariants of multidimensional Riemann metrics related to non-linear differential equations, which have important applications in various areas of modern mathematical physics, will be considered in order to construct their exact solutions. As examples, we consider a 6-dimensional space with the metric

$$^{6}ds^2 = 4U_xpdxdt+4U_ypdydt+(-2pUU_x-2pU_{xxx}-2\mu qU_y+2UU_x)dt^2+$$

$$+2dxdp+2dydq+dUdt,$$

that is Ricci-flat on solutions of the well-known Kadomtsev-Petviashvili equation

$$(U_x)^2+UU_{xx}+U_{xxxx}+U_{xt}+\mu U_{yy}=0$$

and the 14-dimensional Ricci-flat metric on solutions of the Navier-Stokes system of equations, describing the motion of an incompressible viscous fluid [1].

For build new examples of solutions of indicated differential equations the Invariants of E.Cartan [2]

$$S = R(^{i}_{ajb})R(^{j}{}_{cid})k^a k^b k^c k^d, \;\; T = R(^{i}_{ajb;cd})R(^{j}{}_{eif;gh})k^a k^b k^c k^d \, k^e k^f k^g k^h.$$

and $Q = R(_{ab})R(_{cd})k^a k^b k^c k^d$, where $R(^{j}{}_{cid})$ and $R(_{cd})$ are the tensor of Riemann and Ricci-tensor of metric, k^q -components of vector field are applied and their properties are discussed.

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Existence of positive solutions for a semipositone boundary value problem with sequential fractional derivatives

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We investigate the existence and multiplicity of positive solutions for a system of Riemann-Liouville fractional differential equations with sequential derivatives, positive parameters and sign-changing singular nonlinearities, subject to nonlocal uncoupled boundary conditions which contain various fractional derivatives and Riemann-Stieltjes integrals. We apply the nonlinear alternative of Leray-Schauder type and the Guo-Krasnosel'skii fixed point theorem in the proof of our main existence results (see [1]).

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Stability problems of the unperturbed motion governed by the ternary differential system of Lyapunov-Darboux type with nonlinearities of degree four

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We examine the differential system of the unperturbed motion [1,2] with nonlinearities of degree four $s^3(1,4)$, written in the tensorial form [3,4]

$$\dot{x}^j = a^j_\alpha x^\alpha + a^j_{\alpha\beta\gamma\delta}x^\alpha x^\beta x^\gamma x^\delta \ \ (j,\alpha,\beta,\gamma,\delta = 1,2,3) \tag{1}$$

where $a^j_{\alpha\beta\gamma\delta}$ is a symmetric tensor in lower indices in which the total convolution is done. The characteristic equation of this system is

$\varrho^3 + L_1\varrho^2 + L_2\varrho + L_3 = 0$, where the coefficients of this equation are expressed by center-affine invariants $\theta_1 = a^\alpha_\alpha$, $\theta_2 = a^\alpha_\beta a^\beta_\alpha$, $\theta_3 = a^\alpha_\gamma a^\beta_\alpha a^\gamma_\beta$, from [5], and have the form

$$L_1 = -\theta_1, \ \ L_2 = \frac{1}{2}(\theta_2 - \theta_1^2), \ \ L_3 = -\frac{1}{6}(\theta_1^3 - 3\theta_1\theta_2 + 2\theta_3). \tag{2}$$

Using the Lyapunov's theorems on stability of unperturbed and perturbed motion in the first approximation [2], and the Hurwitz's theorem [2], we obtain the following theorems:

Theorem 1. *Assume that the center-affine invariants* (2)*, of the system* (1)*, satisfies the inequalities* $L_1 > 0$, $L_2 > 0$, $L_3 > 0$, $L_1L_2 - L_3 > 0$, *then the unperturbed motion* $x^1 = x^2 = x^3 = 0$, *of the system* (1)*, is asymptotically stable.*

Theorem 2. *If at least one of the center-affine invariant expressions* (2)*, of the system* (1)*, at least one of them with the sign less than zero will be found, then the unperturbed motion* $x^1 = x^2 = x^3 = 0$, *of the system* (1)*, is unstable.*

Theorem 3 [1,2]. *If for the equations of the perturbed motion can be found a function* $V(x) = V(x^1, x^2, x^3)$*, of determined sign, its derivative* $\dot{V}$*, would be of constant sign opposite to the sign of the function* V*, or identical zero, then the unperturbed motion is unstable.*

By a center-affne transformation, the system (1) can be brought to the critical Lyapunov form [1] and in the center-affine conditions

$$\sigma_1 = a^\alpha_\mu a^\beta_\delta a^\gamma_\alpha x^\delta x^\mu x^\nu \varepsilon_{\beta\gamma\nu} \not\equiv 0, \ \eta_1 = a^\alpha_{\beta\gamma\delta\mu} x^\beta x^\gamma x^\delta x^\mu x^\nu y^\theta \varepsilon_{\alpha\nu\theta} \equiv 0,$$

$$L_1, L_2 > 0,$$

the system (1) becomes a critical of Lyapunov-Darboux type, of the form

$$\begin{cases} \dot{x} = -\lambda y + 4xR\,(x,y,z)\,, \\ \dot{y} = \lambda x + 4yR\,(x,y,z)\,, \\ \dot{z} = y - L_1 z + 4zR\,(x,y,z)\,, \end{cases} \tag{3}$$

where $x = x^1, y = x^2, z = x^3$, $R(x,y,z) = a_1x^3 + a_2y^3 + a_3z^3 + 3a_4x^2y + +3a_5x^2z + 3a_6xy^2 + 3a_7xz^2 + 3a_8xyz + 3a_9y^2z + 3a_{10}yz^2$, and a_i $(i = \overline{1,10})$ are coefficients that takes values from the fields of real numbers $\mathbb{R}$.

Using Lie algebra, of the system (3), we obtain the analytic first integral of the form

$$F(x,y,z) \equiv \frac{h_1^3}{(J+h_2)^2} = C \tag{4}$$

governed by the condition

$$J(J+h_2) \neq 0, \tag{5}$$

where

$$h_1 = x^2 + y^2, \quad J = -L_1\lambda^2(4L_1^2 + \lambda^2)(L_1^2 + 4\lambda^2),$$
$$h_2 = \lambda[4(8a_3L_1^2 + 24a_{10}L_1^3 + 12a_5L_1^4 + 24a_9L_1^4 + 8a_2L_1^5 +$$
$$+12a_4L_1^5 - 24a_7L_1^2\lambda - 12a_8L_1^3\lambda + 22a_3\lambda^2 + 66a_{10}L_1\lambda^2 +$$
$$+75a_5L_1^2\lambda^2 + 78a_9L_1^2\lambda^2 + 34a_2L_1^3\lambda^2 + 51a_4L_1^3\lambda^2 -$$
$$-36a_7\lambda^3 - 3a_8L_1\lambda^3 + 18a_5\lambda^4 + 18a_9\lambda^4 + 8a_2L_1\lambda^4 +$$
$$+12a_4L_1\lambda^4)x^3 - 4L_1(12a_7L_1^2 + 12a_8L_1^3 + 8a_1L_1^4 + 12a_6L_1^4 +$$
$$+10a_3\lambda + 30a_{10}L_1\lambda - 24a_5L_1^2\lambda + 24a_9L_1^2\lambda - 12a_7\lambda^2 + 3a_8L_1\lambda^2 +$$
$$+34a_1L_1^2\lambda^2 + 51a_6L_1^2\lambda^2 - 6a_5\lambda^3 + 6a_9\lambda^3 + 8a_1\lambda^4 + 12a_6\lambda^4)y^3 +$$
$$+4a_3\lambda(4L_1^2 + \lambda^2)(L_1^2 + 4\lambda^2)z^3 - 12a_1L_1(4L_1^2 + \lambda^2)(L_1^2 +$$
$$+4\lambda^2)x^2y + 12\lambda(12a_5L_1^4 + 12a_7L_1^2\lambda + 12a_8L_1^3\lambda + 10a_3\lambda^2 +$$
$$+30a_{10}L_1\lambda^2 + 27a_5L_1^2\lambda^2 + 24a_9L_1^2\lambda^2 - 12a_7\lambda^3 + 3a_8L_1\lambda^3 +$$

$$+6a_5\lambda^4 + 6a_9\lambda^4)x^2z + 12(4a_3L_1^2 + 12a_{10}L_1^3 + 12a_9L_1^4 + 4a_2L_1^5 -$$
$$-18a_7L_1^2\lambda - 12a_8L_1^3\lambda + 6a_3\lambda^2 + +18a_{10}L_1\lambda^2 + 24a_5L_1^2\lambda^2 +$$
$$+27a_9L_1^2\lambda^2 + 17a_2L_1^3\lambda^2 - 12a_7\lambda^3 - 3a_8L_1\lambda^3 + 6a_5\lambda^4 + 6a_9\lambda^4 +$$
$$+4a_2L_1\lambda^4)xy^2 + 12\lambda(6a_7L_1^2 + a_3\lambda + 3a_{10}L_1\lambda)(L_1^2 + 4\lambda^2)xz^2 +$$
$$+12L_1\lambda(12a_7L_1^2 + 12a_8L_1^3 + 10a_3\lambda + 30a_{10}L_1\lambda - 24a_5L_1^2\lambda +$$
$$+24a_9L_1^2\lambda - 12a_7\lambda^2 + 3a_8L_1\lambda^2 - 6a_5\lambda^3 + 6a_9\lambda^3)xyz +$$
$$+12\lambda(4a_3L_1^2 + 12a_{10}L_1^3 + 12a_9L_1^4 - 18a_7L_1^2\lambda - 12a_8L_1^3\lambda + 6a_3\lambda^2 +$$
$$+18a_{10}L_1\lambda^2 + 24a_5L_1^2\lambda^2 + 27a_9L_1^2\lambda^2 - 12a_7\lambda^3 - 3a_8L_1\lambda^3 + 6a_5\lambda^4 +$$
$$+6a_9\lambda^4)y^2z + 12L_1\lambda(2a_3 + 6a_{10}L_1 - 3a_7\lambda)(L_1^2 + 4\lambda^2)yz^2].$$

Analyzing the first integral (4), we notice that if the inequality (5) holds, when the function $F(x, y, z)$, from (4), forms *the Lyapunov function.* Then according to the theorem 3, we have

Theorem 4. *If for the system of Lyapunov-Darboux type* (3) *the inequality* (5) *holds, then the unperturbed motion $x = y = z = 0$, governed by this system is stable.*

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Asymptotic behavior of functional stochastic differential equations

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This paper presents some results on the asymptotic equivalence of Functional Stochastic Differential Equations (FSDEs hereafter). Consider the following differential system:

$$dx = f_1(t, x(t))dt \tag{1}$$

Along with the system of FSDEs

$$dy = f_1(t, y(t))dt + f_2(t, y_t)dt + \sigma(t, y_t)dW(t) \tag{2}$$

Where $y_t = y\,(t + \Theta)$, $\Theta \in [-h, 0]$ and $W(t)$ is a Wiener process. It can be shown, that under some assumptions:

1. system (2) is asymptotically mean square equivalent to the system (1) that is for each solution $y(t)$ of system (2) there corresponds a solution $x(t)$ of system (1) such that

$$\lim_{t\to\infty} E|x\,(t) - y\,(t)|^2 = 0$$

2. system (2) is asymptotically equivalent to the system (1) with probability 1 that is for each solution $y(t)$ of system (2) there corresponds a solution $x(t)$ of system (1) such that

$$P\left\{\lim_{t\to\infty}|x(t)-y(t)|=0\right\}=1.$$

Linear differential systems in $\mathbb{R}^3$ having invariant planes with maximal multiplicity

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Let $\mathbb{K}[x,y,z]$ be the ring of polynomials in the x, y and z with coefficients in K, where K is either $\mathbb{R}$ or $\mathbb{C}$. Consider the polynomial differential system in $\mathbb{R}^3$ defined by

$$\dot{x}=P(x,y,z),\ \ \dot{y}=Q(x,y,z),\ \ \dot{z}=R(x,y,z), \tag{1}$$

where P, Q and R are relatively prime polynomials in $\mathbb{R}[x,y,z]$ and the vector field

$$\mathbb{X}=P(x,y,z)\frac{\partial}{\partial x}+Q(x,y,z)\frac{\partial}{\partial y}+R(x,y,z)\frac{\partial}{\partial z}$$

associated to system (1).

Denote $n=\max\{deg(P),deg(Q),deg(R)\}$. If $n=1$ the system (1) is called *linear differential system.*

An *invariant algebraic surface* of differential system (1) or of the vector field $\mathbb{X}$ is an algebraic surface $f(x,y,z)=0$, with $\mathbb{X}(f)=\langle(P,Q,R),\nabla f\rangle=Kf$, where ∇f denotes the gradient of the function f. The polynomial K is called the *cofactor* of the invariant algebraic surface $f=0$ and if m is the degree of the vector field $\mathbb{X}$, then the degree of K is at most $m-1$. If the polynomial f is irreducible in $\mathbb{C}[x,y,z]$, then we say that f is an *irreducible invariant algebraic surface*. If the degree of f is 1 then we say that the invariant algebraic surface f is an *invariant plane*.

Let W be a finite $\mathbb{R}$ - vector subspace of $\mathbb{R}[x,y,z]$ such that $dim(W) = N$ and $\{v_1, v_2, ..., v_N\}$ be a basis of W. The *extractic polynomial of* $\mathbb{X}$ *associated to* W is the polynomial

$$E_W(\mathbb{X}) = det \begin{pmatrix} v_1 & v_2 & ... & v_N \\ \mathbb{X}(v_1) & \mathbb{X}(v_2) & ... & \mathbb{X}(v_N) \\ ... & ... & ... & ... \\ \mathbb{X}^{N-1}(v_1) & \mathbb{X}^{N-1}(v_2) & ... & \mathbb{X}^{N-1}(v_N) \end{pmatrix} \qquad (2)$$

where $\mathbb{X}(v_i) = \langle (P,Q,R), \nabla v_i \rangle$ and $\mathbb{X}^{k+1}(v_i) = \mathbb{X}(\mathbb{X}^k(v_i))$, for $k = \overline{1, N-2}$. If $W = \mathbb{R}_m[x,y,z]$, where $\mathbb{R}_m[x,y,z]$ in the $\mathbb{R}$ - vector subspace of polynomials in $\mathbb{R}[x,y,z]$ of degree at most m, we say that the polynomial $E_W(\mathbb{X})$ is the $m-th$ *extractic polynomial* of $\mathbb{X}$ and denote it by $E_W^m(\mathbb{X})$. We say that an irreducible invariant algebraic surface $f = 0$ of degree m has *algebraic multiplicity* k if $E_W^m(\mathbb{X}) \not\equiv 0$ and k is the maximum positive integer such that f^k divides $E_W^m(\mathbb{X})$; and we say that it has *no defined algebraic multiplicity* if $E_W^m(\mathbb{X}) \equiv 0$.

In this work we show that in the class of linear differential systems

$$\begin{cases} \dot{x} = a_{000} + a_{100}x + a_{010}y + a_{001}z, \\ \dot{y} = b_{000} + b_{100}x + b_{010}y + b_{001}z, \\ \dot{z} = c_{000} + c_{100}x + c_{010}y + c_{001}z \end{cases} \qquad (3)$$

having invariant planes with maximal multiplicity.

Theorem. *Via an affine transformation of coordinates and time rescaling each the linear differential systems in* $\mathbb{R}^3$ *having invariant planes of their maximum multiplicities can be written in one of the forms:*

1. $\dot{x} = x, \dot{y} = b_{000} + b_{100}x + y + b_{001}z, \dot{z} = c_{000} + c_{100}x + z,$
$b_{001}c_{100} \neq 0;$
2. $\dot{x} = x, \dot{y} = b_{010}y, \dot{z} = c_{000} + c_{100}x + c_{010}y + z,$
$b_{010}c_{100}(b_{010} - 1) \neq 0;$
3. $\dot{x} = x, \dot{y} = b_{010}y, \dot{z} = c_{001}z,$
$b_{010}c_{001}(b_{010} - 1)(b_{010} - c_{001})(c_{001} - 1) \neq 0.$

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A qualitative study of the quintic differential system with maximal multiplicity of the line at the infinity

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Consider the generic quintic differential system. In [1] it was proven that the class of differential quintic systems that have the invariant straight line at the infinity of maximal multiplicity is affine equivalent with the system

$$\begin{cases} \dot{x} = x, \quad a \in \mathbb{R}^*, \\ \dot{y} = -4y + ax^5. \end{cases} \tag{1}$$

In this paper, by using several techniques, we obtain its phase portrait on the Poincaré disk.

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Invariant Measures and Asymptotic Behavior of Stochastic Evolution Equations

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We consider the stochastic evolution equation in Hilbert space H

$$\begin{cases} du = (Au + F(u))dt + B(u)dW(t); \\ u(0) = u_0 \in H. \end{cases} \tag{1}$$

Here A is an elliptic operator, F and B are nonlinear maps, and the Gaussian noise $\dot{W}(t,x)$ is white in time and colored in space, defined below.

The evolution equations of this type, both deterministic (with $B \equiv 0$) and stochastic, emerged in the first half of the 20th century as models of interplay between diffusion and reaction terms. The range of application of (1) includes the population dynamics, chemical physics, biomedical modeling, modeling consumption of resources etc.

We assume the operator and the nonlinearities satisfy the following conditions:

[i] A is the infinitesimal generator of strongly continuous semigroup $S(t) = e^{tA}, t \geq 0$ in H.

[ii] The map $F : H \to H$;

[iii] The mapping $B : H \to \mathcal{L}_2^0(U_0, H)$, where $\mathcal{L}_2^0(U_0, H)$ is the space of Hilbert-Schmidt operators $U_0 := Q^{1/2}U$ to H.

A predictable H-valued process $u(t), t \in [0, T]$ is a mild solution of (1) if

$$P\left\{\int_0^t \|u(s)\|^2 ds < \infty\right\} = 1,$$

and for any $t \in [0, T]$ we have

$$u(t) = S(t)u_0 + \int_0^t S(t-s)F(u(s))ds + \int_0^t S(t-s)B(u(s))dW(s). \quad (2)$$

If A satisfies [i], F and B are Lipschits and grow at most linearly, the initial value problem (1) has a unique mild solution, which is a stochastically continuous homogeneous Markov process, which also satisfies the Feller property.

Denote $M(H)$ to be the set of probability measures on H. An element $\mu \in M(H)$ is called an invariant measure for the Markov semigroup P_t if

$$\int_H \varphi(v_0)d\mu(v_0) = \int_H P_t\varphi(v_0)d\mu(v_0).$$

We present sufficient conditions for the existence of invariant measures. These conditions are expressed in terms of the coefficients of the equations.

Classification of cubic differential systems with a linear center and the line at infinity of maximal multiplicity

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We consider the real cubic system of differential equations

$$\begin{array}{l} \dot{x} = y + ax^2 + cxy + fy^2 + kx^3 + mx^2y + pxy^2 + ry^3 \equiv p\,(x,y)\,, \\ \dot{y} = -(x + gx^2 + dxy + by^2 + sx^3 + qx^2y + nxy^2 + ly^3) \equiv q\,(x,y)\,, \\ \gcd(p,q) = 1, sx^4 + (k{+}q)x^3y + (m{+}n)x^2y^2 + (l{+}p)xy^3 + ry^4 \not\equiv 0 \end{array} \tag{1}$$

and the homogeneous system associated to the system (1): $\{\dot{x} = P\,(x,y,Z)\,,\ \dot{y} = Q\,(x,y,Z)\}$, where

$$\begin{array}{l} P\,(x,y,Z) = yZ^2 + (ax^2 + cxy + fy^2)Z + kx^3 + mx^2y + pxy^2 + ry^3, \\ Q\,(x,y,Z) = -(xZ^2 + (gx^2 + dxy + by^2)Z + sx^3 + qx^2y + nxy^2 + \\ \quad + ly^3). \end{array}$$

Denote $\mathbb{X}_\infty = P\,(x,y,Z)\,\frac{\partial}{\partial x} + Q\,(x,y,Z)\,\frac{\partial}{\partial y}$.

The linearized system (1) in critical point $(0,0)$ has a center in this point, i.e. $(0,0)$ is a linear center for (1).

We say that for (1) the line at infinity $Z = 0$ has *multiplicity* $\nu + 1$ if ν is the greatest positive integer such that Z^ν divides $E_\infty = P\mathbb{X}_\infty(Q) - Q\mathbb{X}_\infty(P))$[1].

Theorem 1. *In the class of cubic differential systems of the form (1) the maximal multiplicity of the line at infinity is five.*

Theorem 2. *The system* (1) *has the line at infinity of multiplicity five if and only if its coefficients verify one of the following three set of conditions:*

$$C = D = 0,\ B = -AS^3/K^3,\ F = -AS^2/K^2,\ G = AS/K,\ M = S,\\ L = -S^4/K^3,\ N = R = -S^3/K^2,\ Q = -P = S^2/K; \tag{2}$$

$$A{=}5F^3/B^2, C{=}{-}6F^2/B, D{=}2F, G{=}{-}3F^2/B, K{=}F^5/B^3, L{=}BF,\\ M{=}{-}3F^4/B^2, N{=}{-}3F^2, P{=}Q{=}3F^3/B, R{=}{-}F^2, S{=}{-}F^4/B^2,\ S \neq 0; \tag{3}$$

$$A = -K^2(2BK - 3FS)/S^3, C = -2K(BK - 2FS)/S^2, D = 2F,\\ G = K(2FS - BK)/S^2, L = S^4/K^3, M = 3S, N = 3S^3/K^2,\\ P = 3S^2/K, Q = 3S^2/K, R = S^3/K^2,\ K^2(BK - FS)^2 + 4S^5 = 0, \tag{4}$$

where

$$A = g - b - c + i(a + d - f),\ C = 2(b + g + i(a + f)),\\ F = c + g - b + i(a - d - f),\ K = r + s - m - n + i(k - l - p + q),\\ M = n - m - 3(r - s) + i(3(k + l) + p + q),\\ P = m + n + 3(r + s) + i(3(k - l) + p - q),\\ R = m - n - r + s + i(k + l - p - q),\\ B = \overline{A},\ D = \overline{C},\ G = \overline{F},\ L = \overline{K},\ N = \overline{M},\ Q = \overline{P},\ S = \overline{R}, \tag{5}$$

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The connection between oscillation of solutions of linear equations and their corresponding equations on time scales

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Time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. Assume $\mathbb{T}$ has the topology that it inherits from the real numbers $\mathbb{R}$ with the standard topology.

As object of our research is oscillation of solutions dynamic equations, then we will consider that $\sup\mathbb{T} = \infty$. For every interval $[a, b] \subset R$ we will define the interval $[a, b]_{\mathbb{T}} = [a, b] \cap \mathbb{T}$.

For every $t \in \mathbb{T}$ defined [1] three functions that characterize the scale: the forward jump operator $\sigma(t) := inf\{s \in \mathbb{T} : s > t\}$, the backward jump operator $\rho(t) := \sup\{s \in \mathbb{T} : s < t\}$ and the graininess function $\mu : \mathbb{T} \to [0; 1)$ such that $\mu(t) := \sigma(t) - t$. If $t > inf\,\mathbb{T}$ and $\rho(t) = t$ then point t is called right-dense, and if $t < \sup\mathbb{T}$ and $\sigma(t) = t$ then t is called left-dense. Points that are right-dense and left-dense at the same time are called dense. Also, if $\sigma(t) > t$, we say that t is right-scattered, while if $\rho(t) < t$ we say that t is left-scattered. Points that are right-scattered and left-scattered at the same time is called isolated.

We consider problem of finding connection between oscillation of solutions of second-order linear differential equation on interval $[0, a]$

$$\ddot{x} + p(t)\dot{x} + q(t)x == 0, \tag{1}$$

where $p, q{\in}C\left([0,a]\right)$ and oscillation of solutions of relevant dynamic equation on set of time scales $\mathbb{T}_\lambda$

$$x_\lambda^{\Delta\Delta} + p(t)x_\lambda^{\Delta} + q(t)x_\lambda = 0, \tag{2}$$

where $x_\lambda : \mathbb{T}_\lambda \to \mathbb{R}^d$, and $x_\lambda^{\Delta}\left(t\right)$ is delta-derivative of $x\left(t\right)$ on $\mathbb{T}_\lambda$. We assume that $inf\,\mathbb{T}_\lambda = -\infty$, $\sup \mathbb{T}_\lambda = \infty$, $\lambda{\in}\Lambda{\subset}R$, and $\lambda = 0$ is a limit point of Λ.

Denote $\mu_\lambda := \sup\limits_{t\in\mathbb{T}_\lambda} \mu_h\left(t\right)$, where $\mu_\lambda : \mathbb{T}_\lambda \to [0,\infty)$ is the graininess function. It is straightforward to see that if $\mu_\lambda\left(t\right) \to 0$ as $\lambda \to 0$, then $\mathbb{T}_\lambda$ coincides (e.g., in Hausdorff metric) with a continuous time scale $\mathbb{T}_0 = R$.

Definition 1. *We say that the solution $x_\lambda\left(t\right)$ of the dynamic equation (2) has a generalized zero at t if $x_\lambda\left(t\right) = 0$ or, if t is right-scattered and $x_\lambda\left(t\right) \cdot x_\lambda\left(\sigma\left(t\right)\right) < 0$.*

Definition 2. *Solution of the differential equation (1) and solution of the dynamic equation (2) are called relevant each other if they have the same initial data.*

Definition 3. *If the solution $x\left(t\right)$ of the dynamic equation (2) have not less than two generalized zeros at certain interval that we will called it oscillatory at that interval.*

We received two theorems.

Theorem 1. *If the solution $x\left(t\right)$ of the differential equation (1) oscillatory at $[0,a]$ then for small enough μ_λ relevant solution $x_\lambda\left(t\right)$ of the dynamic equation (2) also oscillatory.*

Theorem 2. *If the solution $x_\lambda\left(t\right)$ of the dynamic equation (2) oscillatory at $[0,a]_{\mathbb{T}_\lambda}$ then for small enough μ_λ relevant solution $x\left(t\right)$ of the differential equation (2) also oscillatory.*

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About boundary value problems on propagation of non-stationary longitudinal waves in rods of variable cross section

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Based on the application of the integral variational principle of Hamilton-Ostrogradsky with the use of isochronous variations, the statements of dynamic boundary value problems on the propagation of non-stationary waves in rods of variable cross section are obtained. Two displacement models have been developed: single-mode (technical) and two-mode (taking into account transverse movements). The kinetic energy of transverse motions and the shear strain energy are taken into account. The obtained statements of boundary value problems are reduced to a dimensionless form for the special case circular section, exponentially changing with the longitudinal coordinate. It is noted that natural boundary conditions are more complicated than those obtained using the formal application of Hooke's uniaxial law. The formulated problems are solved by the method of characteristics, taking into account the dependence wave equation coefficients on the longitudinal coordinate.

Variational and PDE-based mixed Poisson-Gaussian noise removal techniques

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Digital images are often corrupted by noise mixtures than can be modeled as combinations of Gaussian and Poisson distributions, which are usually generated by the acquisition devices. Various effective mixed Poisson-Gaussian noise filtering approaches have been developed in the last years. Those state of the art mixed noise removal techniques that use nonlinear partial differential equation (PDE) based models are surveyed here. Our own contributions in this image restoration domain are also described in this work.

Several PDE variational models for mixed Poisson-Gaussian noise reduction, which use the total variation regularization, are presented first. Some of them combine the TV - ROF Denoising model that removes the Gaussian noise to modified TV schemes, adapted for the Poisson noise. Other variational mixed noise removal solutions are based on spatially adaptive total variation regularization terms. A fast total variation-based algorithm for image restoration with exact Poisson–Gaussian likelihood is also disscused in this work. Another variational Poisson-Gaussian denoising technique presented here uses a functional combining the TV regularisation, Kullback-–Leibler divergence and the L2 norm. Then, second-order total generalized variation (TGV) regularized mixed noise removal models are described. These variational denoising techniques lead to some nonlinear PDE models that are next discretized by applying some finite difference-based numerical algorithms.

Then, several PDE-based restoration approaches dealing with this noise mixture and proposed by us are described here. Besides a variational Poisson-Gaussian noise reduction method, we have also developed some non-variational mixed noise removal techniques based on well-posed PDE models. One of them uses a parabolic nonlinear fourth-order PDE model, while another method presented here is based on a second-order hyperbolic PDE model. Method comparison results illustrating their effectiveness are also provided.

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Vectors and eigenvalues when calculating the dipole strength

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This work is centered on some results obtained based on studies done on papers [4]. The ref. [4] describes the idea that when studying the vibration modes of molecules with infrared intensities (IR) only three intensities are used, as described by three eigenvectors, because the others are zero. We are interested in determining the vectors and eigenvalues of a matrix according to the scheme in the paper [4] only for VCD (vibrational circular dichroism) technology and to establish a link between vectors, eigenvalues and dipole strength.

We will denote by $\vec{\mu}_E = [\vec{E}_{01}(i)]_\beta$ (EDTM -electric dipole transition moments) associated with the fundamental transition $|0> \longrightarrow |1>$ of the i^{th} vibrational mode, where β are the Cartesian coordinates for atoms, $i = \overline{1, 3N}$ are vibrational mode. Let the matrix:

$$M_{pq} = \sum_{k=1}^{3} \frac{\partial \vec{\mu}_k}{\partial Q_p} \frac{\partial \vec{\mu}_k}{\partial Q_q}, \tag{1}$$

where $\frac{\partial \vec{\mu}_k}{\partial Q_p}$ is the $k-th$ component of the dipole derivative (of mass-weighted) with respect to normal coordinate Q_p, $1 \leq p$, $q \leq N$.

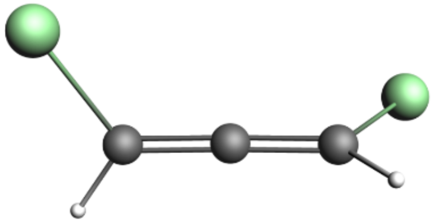

Figure 6. The molecule of 1,3-dichloroallene, 7 atoms

We diagonalize the matrix M from eq. (1) we will obtain only three non-zero values, so that we can study the vibration modes only according to the vectors corresponding to the eigenvalue. A connection was also established between the eigenvectors corresponding to t_j eingenvectors T_j and the dipole strengh (D_{01}) (Fig.1).

$$\sum_{i=1}^{3N} D_{01}(i) = \sum_{j=1}^{3N} t_j T_j^2. \tag{2}$$

The above study makes it possible to study the spectra and vibrations of molecules not only with experimental technologies but also studying with technologies of computational physics based on vibrational circular dichroism (VCD).

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Use of the mathematical method for determining fire risk

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This paper describes the procedure for performing an analysis on fire risk identification for a civil building. For the process of estimating and quantifying the risk associated with a system for evaluation, I used the mathematical method. The risk and protection factors used were in line with the main provisions of the national fire safety regulations. The article determines the calculation relationships for the fire risk (relative to the accepted risk for the type of objective considered) and for fire safety, as well as the expression of the terms used.

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A cyberinfrastructure-driven big data analysis for air pollution and human health

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Air pollution in urban areas is one of the major environmental challenges affecting significantly the health of inner citizens. The existing monitoring and data processing infrastructure applied today does not assess completely and timely the micro-scale dynamics of the air pollution processes. In the last decade, two directions were used for a successful increasing of information amount regarding the air pollutants in urban microenvironments to complement and supplement the information from authorities? conventional infrastructure: (i) implications of citizens as observers of air pollution including Volunteered Geographic Information (VGI), and (ii) developing of intelligent cyberinfrastructures. The current approach presents an intelligent VGIS- and crowdsourcing- enabled cyber-structure and the associated air pollution forecasting with a wider scope for citizens, businesses, urban planners, academics, other stakeholders and decision-makers regarding the personal protection against outdoor air pollution episodes.

A critical case for stability in a model of an electrohydraulic servomechanism

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In this paper, the conditions required for the stability of a nonlinear system with time-delay and switching are studied. The starting point of the theory is based on a real-world mathematical model of an electrohydraulic servomechanism located in ailerons flight controls of the Romanian IAR 99 Hawk jet training airplane. For this model, a general theorem on the equilibrium stability in a critical case for a switched nonlinear system of delay differential equations is stated. The framework uses multiple complete Lyapunov-Krasovskii functionals. The characteristic equation has one zero root which claims the use of a special approach given by a Lyapunov-Malkin theorem. Therefore, some transformations are made to write the linearized system in a canonical form where the stability Lyapunov theorem of the linear approximation can be applied. The equilibrium conditions requires the fulfilment of two requisites: a Lyapunov condition and an asymptotic stability condition. The transformation of the nonlinear system into the specific form of the Lyapunov-Malkin theorem and the verification of the two conditions mentioned above requires a double perspective – analytical developments and numerical simulations – since the mathematical models are too complex to be approached only analytically. Accordingly, an important result is calculated, regarding an admissible delay threshold in preserving stability of the electrohydraulic servomechanism as vital system for the safety of the aircraft. Some considerations regarding the conservatism and the non-necessity of sufficient conditions conclude the work.

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Statistical aspects regarding some physical sources of noise in electronic devices

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Three types of random noise are analyzed statistically and spectrally: "shot" noise, generation / recombination noise, avalanche noise. The noise power spectral density is calculated. A system generating a signal but with fluctuating excitation is described by differential equations with random terms. The general mathematical framework of random processes provides a unitary interpretation for spectral analysis of noise.

Dam Break Flow with Erosion and Vegetation

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We consider a mathematical problem of theoretical and practical interest: the dam break flow on an erodible and vegetated soil-bed. The problem is to find the solution of the coupled shallow water equations and Hairisin-Rose erosion model in the presence of vegetation, [1].

Theoretically, one needs to find singular solutions required by the physical model and to solve some problems generated by the presence of discontinuity in coefficients of the PDE mathematical model.

From a practical point of view, the problem is of large interest for hydrologists and officials responsible with the management of water resources.

In this paper, we focus on numerical solutions of some benchmark dam break problems, [2, 3, 4].

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Some evaluations for the mathematical model's control associated to Kuznetsov-Makalkin immunogenic model

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The task of mathematical modeling the cancer is a monumental challenge. Past and current mathematical models have been proposed to examine the tumor growth and the discussions and challenges still remain. The present paper concerns with a basic model proposed by Kuznetsov, Makalkin and Taylor for immunogenic tumors. Another standpoint is taken into account, namely, the existence of a state space exact linearization for the dynamics of the model is approached. The basic 2d case is tested for the moment. The important result is that the 2d model admits a state space exact linearization, in feasible conditions for the parameters. The paper continues some recent work on this subject. The control u involved in the method can have a graphical evaluation, based on some parameters values from the literature and experiments. Some useful evaluations of it are done using the Maple Software. The analysis would be useful in further analysis, together with the extension to the 3d model.

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Numerical simulation of transient processes in a heat exchanger

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We consider the possibilities of numerical simulation for modelling the dynamic processes in a heat transferring device. The principle of operation of considered tube-in-tube heat exchanger is based on the constant contact of the coolant with the treated liquid. The mathematical model of the transient process of transferring heat energy in devices of this type includes a system of three differential equations for the temperatures of cold water (heated), hot water (heating) and dividing walls with corresponding initial and boundary conditions.

The numerical algorithm based on the ideas of finite difference method is elaborated. The issue of convergence of the constructed difference scheme based on the study of its approximation and stability is considered. The approximation is easily proved by expanding discrete functions in a Taylor series and, for the problem under consideration, is of the first order with respect to the grid steps. The stability of the constructed difference scheme is proved using the Neumann spectral criterion.

Simulation of the interaction of real gas and walls with different initial temperatures

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This paper studies the flow arising from the conjugate interaction of a flow of real gas at rest and a heat-conducting stationary impermeable wall, which initial temperature differs from the gas initial temperature. Within the conjugate approach, gas-dynamic and thermal processes lead to emergence of a flow with a complex internal structure and formation of certain characteristic zones in the flow field. The wide distribution of these processes in nature and technology implies the necessity to study the effects, accompanying the interaction of gas flows with various barriers, which is of considerable theoretical and applied interest. A thorough experimental study of interaction processes is associated with significant difficulties, and therefore the use of mathematical modelling methods for the studied dynamic processes has become a powerful tool in recent years, allowing to solve various problems of continuum mechanics and study the occurring phenomena. The concept of an approximate method for studying the emerging flow based on the complete system of Navier-Stokes equations, which successfully models the laws of conservation of mass, momentum and energy in a wide range of gas parameters, has been developed and theoretically grounded. The temperature distribution on the wall is modelled by the linear heat conduction equation.

The set of the Navier-Stokes equations for gas and the thermal conductivity equation for the wall, with given initial and boundary conditions, represents the mathematical model for studying dynamics of the emerging flow of real gas in the presence of the initial temperatures jumps of gas and the wall. An approximate solution of the problem is given for moderate initial temperatures jumps, when a flow with small perturbations of the parameters is formed. In this case, an approximate approach is used based on the representation of each gas-dynamic parameter as a sum of two quantities, the first of which corresponds to the value of the parameter in the initial state, and the second expresses a small non-stationary perturbation. Taking into account small perturbations of the parameters, the Navier-Stokes equations are linearized around the values of the parameters in the initial state. An analytical solution of the linearized problem is obtained by the Laplace integral transforms method in the form of integral representations of the parameters of a complex variable with the Prandtl number (Pr) equal to one. The obtained solution describes the main characteristic features of the formation of a continuous structure of the gas flow field during its interaction with a heat-conducting cold or hot wall. In this case, it is possible to evaluate the influence of viscosity, thermal conductivity, accommodation, and other physical factors on the formation of dissipative and ideal inviscid and non-heat-conducting zones in the gas flow field.

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Impact of social influence on student performance and dropout: a mathematical modelling approach

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We aim to determine the influence of social and cognitive factors such as self-efficacy, locus of control and exposure to negative social and peer influence on dropout intentions and academic performance of undergraduate students. To this purpose, we first consider the nature of mechanisms of social influence and subsequently, influenced by epidemiological concepts and considerations, introduce and analyze a compartmental model consisting in a system of non-linear ODEs. Via the next generation approach, we find threshold values to be understood as reproduction numbers, that govern the stability of the equilibria. These threshold values can be viewed as target values to be reached in order to alleviate undergraduate students dropout. A backward bifurcation is observed to occur, analytically and numerically, provided that certain conditions are satisfied.

A sensitivity analysis is then performed to find how the threshold values respond to changes in the parameters. Concrete values for these parameters are then computed using survey data from a Ghanaian university. Finally, our findings are interpreted from a social cognitive perspective, realistic policy changes being proposed along with appropriate teaching and coaching strategies.

4. Real, Complex, Functional and Numerical Analysis

Amenable quasi-lattice ordered groups

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Let (G, P) be a quasi-lattice ordered group. In previous work, the author constructed a universal covariant representation (A, U) for (G, P) in a way that avoids some of the intricacies of the other approaches in [2] and [3]. Then showed if (G, P) is amenable, true representations of (G, P) generate C^*-algebras which are canonically isomorphic to the C^*-algebra generated by the universal covariant representation. In this paper, we discuss characterizations of amenability in a comparatively simple and natural way to introduce this formidable property

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On unbounded order weakly demicompact operators

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A Banach space X is called a Banach lattice if it is a vector lattice and the norm satisfies the property that if $\mid x \mid\leq\mid y \mid$ implies $\| x \|\leq\| y \|$ for every $x, y \in X$. An operator $T : X \to X$ is called demicompact if, for every bounded sequence (x_n) in X such that $(x_n - Tx_n)$ converges in X , then there is a convergent subsequence of (x_n).

In this talk, our aim is to use the theory of Banach lattices to provide an approach to the unbounded order weakly demicompact operators. We char- acterize Banach lattices on which all operators are unbounded order weakly demicompact.

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Triangular ratio metric and Ptolemy-Alhazen problem

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The study of hyperbolic-type intrinsic metrics, such as the triangular ratio metric, revived the interest for the Ptolemy-Alhazen (P-A) problem, an ancient problem that asks to find on a given mirror the point, called the P-A point, where the light from a given pointwise source will be reflected to the eye of an observer. Recently, Ptolemy-Alhazen problem has been applied to Computer Vision [1], [2] and to Astrophysics [5]. Our approach to P-A problem adopts algebraic and geometric viewpoints, for spherical (concave or convex) mirrors, as well as for general quadric mirrors. In the case of a spherical mirror we refind a self-inversive quartic equation satisfied by the affix of the P-A point in a certain plane, using at least three methods. In the general case of a quadric surface mirror, we show that the affix of the P-A point satisfies a sixth-degree algebraic equation. We discuss three geometric approaches applicable for spherical mirrors, involving respectively a hyperbola, Apollonian circles and the catacaustic of a circle. We also discuss applications of the triangular ratio metric to geometric function theory.

This talk is based on joint works with Masayo Fujimura, Parisa Hariri and Matti Vuorinen [3], [4].

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Aggregation operators with applications in Mathematical economics

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We present some aggregation operators based on integrals (as the Choquet-like, Sugeno-like integrals) which will be used in the Mathematical economics models for defining the feasible allocations and the core. For this models we study the „welfare theorems" and the „Edgeworth conjecture", the foundamental results from the General Equilibrium Theory.

Invariant measure for stochastic functional differential equations

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In this work we study the asymptotic behaviour of the solutions of neutral type stochastic functional-differential equations of the form

$$d[u(t)+g(u_t)]=[Au+f(u_t)]dt+\sigma(u_t)dW(t) \text{ for } t>0; \tag{1}$$
$$u(t)=\varphi(t), t\in[-h,0),\ \ h>0.$$

Here A is an inifinitesimal generator of a strong continuous semigroup $\{S(t), t\geq 0\}$ of bounded linear operators in real separable Hilbert space H. The noise $W(t)$ is a Q-Wiener process on a separable Hilbert space K. For any $h>0$ denote $C_h := C([-h,0],H)$ to be a space of continuous H-valued functions $\varphi:[-h,0]\to H$, equipped with the norm

$$\|\varphi\|_{C_h} := \sup_{t\in[-h,0]} \|\varphi(t)\|_H,$$

where $\|\cdot\|_H$ stands for the norm in H. The functionals f and g map C_h to H, and $\sigma: C_h\to\mathcal{L}_2^0$, where $\mathcal{L}_2^0=\mathcal{L}(Q^{1/2}K,H)$ is the space of Hilbert-Schmidt operators from $Q^{1/2}K$ to H. Finally, $\varphi:[-h,0]\times$

$\Omega \to H$ is the initial condition, where $(\Omega, \mathcal{F}, P)$ is the probability space.

We study two questions: the existence and uniqueness of the solution to the initial problem and establish the existence of invariant measures in the shift spaces for such equations. Our approach is based on Krylov-Bogoliubov theorem on the tightness of the family of measures. We present sufficient conditions for the existence of invariant measures. These conditions are expressed in terms of the coefficients of the equations.

On spline-collocation and spline-quadratures algorithms for solving integral and weak-singular integral equations of second kind

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The work includes algorithms and substantiation theory in some Banach space of spline-collocation and spline-quadratures methods in solving the following integral equations of second kind:

1) Fredholm linear integral equations of second kind;
2) Volterra linear integral equations of second kind;
3) Fredholm linear weak-singular integral equations of second kind;
4) Volterra linear weak-singular integral equations of second kind.

For proposed integral equations, new spline-collocation and spline-quadratures algorithms are developed, such as (see Abstracts of CAIM-2017, CAIM-2018 and CAIM-2019):

1) spline-collocations algorithms for solving Fredholm and Volterra linear integral equations of second kind, which use as basic functions some convex and concave splines;
2) spline-quadratures algorithms for solving Fredholm and Volterra linear integral equations of second kind, which use as basic functions the same convex and concave splines;
3) spline-collocations algorithms for solving Fredholm and Volterra linear weak-singular integral equations of second kind, which use as basic functions the linear and some convex and concave splines;
4) spline-quadratures algorithms for solving Fredholm and Volterra linear weak-singular integral equations of second kind, which use as basic functions the linear and some nonlinear splines.

Following, we have established sufficient conditions on compatibility and convergence of the developed computing algorithms in spaces of continuous functions and Hölder spaces.

5. Probability Theory, Mathematical Statistics, Operations Research

Extreme values modeling using the Gamma-Lognormal-Pareto three-spliced distribution

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Extreme value theory (EVT) is a field of probability and statistics dealing with models for extreme values. The study of extreme events is very important because, even if they rarely happen, extreme events can have a catastrophic impact once they happened. Therefore, EVT has applications in many fields such as climatology, hydrology, engineering, traffic prediction, finance, insurance, epidemics etc.

When data exhibit high frequency of small to medium values and low frequency of large values, fitting a classical distribution might fail. This is why in the recent years, the study of spliced models gained a lot of interest in the development of univariate EVT models. This interest is due to the particular form of spliced models, which are defined from different distributions on distinct adjacent intervals. This particularity insures a better fit to specific data presenting extreme values.

In contrast to the intensive study of two-spliced distributions, the case with more than two components is scarcely approached. In this paper, we introduce and study the three-spliced Gamma-Lognormal-Pareto distribution. A special attention is paid to the estimation procedure, especially since the thresholds where such distributions change shape are considered unknown parameters, which makes the estimation more challenging. We illustrate the estimation procedure on simulated data.

Convex covering problem of graphs resulting from some graph operations

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We present new findings related to the convex covering problem of graphs resulting from some graph operations.

Convex covers of graphs are defined in [1]. In general, convex covering problem of graphs is NP-complete [2]. So, it is of interest to investigate this problem for graphs resulting from different graph operations.

We determine the maximum nontrivial convex cover number of the join and corona of graphs. These results are partially presented in [3].

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On convexity threshold and the coalitional rationality thresholds for cooperative transferable utilities games

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This paper summarizes the results obtained in connection to the following problems:
(A) For a cooperative game in which the Shapley Value is not coalitional rational, find out a new game with the same value, in which the value is coalitional rational.
(B) For a cooperative game which is not convex, find out a new game in which the Shapley Value is the same, but the game is convex.
(C) The same problems about coalitional rationality and convexity, for two other values, the Egalitarian Allocation and the Egalitarian Nonseparable Contribution, in a new game where these values are unchanged. The convexity threshold and the coalitional rationality thresholds have been introduced and their relationships have been determined. We shall try to discuss the main ideas and give some numerical illustrations.

Parallel algorithm to solving block-cyclic partitioned bimatrix games

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We consider the bimatrix game in the following strategic form $\Gamma = \langle I, J, A, B\rangle$ and denote by $NE[\Gamma]$ the set of all equilibrium profiles in the game Γ. Matrices A and matrix B are divided into submatrices using one and the same algorithm. Thus we can obtain a series of pairs submatrices of the same size $\{(A_r, B_r)\}_{r=\overline{1,p}}$ where $A_r = \left\|a_{ij}^r\right\|_{i\in I_r}^{j\in J_r}$, $B_r = \left\|b_{ij}^r\right\|_{i\in I_r}^{j\in J_r}$ and $I_{r_1} \cap I_{r_2} = \oslash$, $J_{r_1} \cap J_{r_2} = \oslash$ $\forall r_1 \neq r_2$. Here the index r actually means "processor" which, will obtain these submatrices. These submatrices will generate a series of games which are actually sub-games of the original game $\Gamma_r = \langle I_r, J_r, A_r, B_r\rangle$. We denote by $NE[\Gamma_r]$ the set of Nash equilibrium profiles in the problem Γ_r and assume that the subgames are solved in parallel on an HPC system. If for any $(i_r^*,\ j_r^*) \in NE[\Gamma_r]$ we have that $(i_r^*,\ j_r^*) \in NE[\Gamma]$ then we will say that the algorithm of dividing the matrices into blocks of submatrices is perfect and will be called perfect matrix dividing and distribution (PMDD) algorithm.

According to the two dimensional block cyclic data distribution algorithm [1] all process can be referenced by its row and column coordinates, (l, c) and must solve the $\Gamma_{(c,l)} = \left\langle I_{(c,l)}, J_{(c,l)}, A_{(c,l)}, B_{(c,l)}\right\rangle$ game. We can proof the folowing theorem.

Theorem *If*

1. *for fixed* c *and all* $\widetilde{l} \neq l$ *such that* $(\widetilde{l}, c) \in L \times C$ *the conditions* $a_{i^*_{(l,c)}, j^*_{(l,c)}} \geq a_{i^*_{(\widetilde{l},c)} j^*_{(l,c)}}$ *are fulfilled;*

2. for fixed l and all $\widetilde{c} \neq c$ such that $(l, \widetilde{c}) \in L \times C$ the conditions $b_{i^*_{(l,c)}, j^*_{(l,c)}} \geq b_{i^*_{(l,c)} j^*_{(l,\widetilde{c})}}$ are fulfilled.

Then the two dimensional block cyclic data distribution algorithm is a PMDD algorithm.

Here $i^*_{(\widetilde{l},c)} = \arg \max\limits_{i_{(\widetilde{l},c)} \in I_{(\widetilde{l},c)}} a_{i_{(\widetilde{l},c)} j^*_r}$ and $j^*_{(l,\widetilde{c})} = \arg \max\limits_{j_{(l,\widetilde{c})} \in J_{(l,\widetilde{c})}} b_{i^*_r j_{(l,\widetilde{c})}}$.

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Parallel algorithm for mixed HPC systems to determine the solutions in the bimatrix informational extended games

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According to [1] we can construct and implement on HPC systems the following parallel algorithm to find the all informational extended equilibrium profiles in bimatrix games.

Algorithm

1. Using the MPI programming model we generate the MPI Communicator with linear topology and dimension $\varkappa_1 \cdot \varkappa_2$. Root process, using the `MPI_Bcast` function, broadcasts to all MPI process the initial matrices $A = ||a_{ij}||_{i \in I}^{j \in J}$, and $B = ||b_{ij}||_{i \in I}^{j \in J}$ of the bimatrix game $\Gamma = \langle A, B \rangle$.

2. Using the MPI programming model and open source library ScaLAPACK-BLACS, the processes grid $\{(\alpha,\beta)\}_{\alpha=\overline{1,\varkappa_1}}^{\beta=\overline{1,\varkappa_2}}$ is initialized, and in parallelall fixed $(\alpha,\beta)-$processes, using combinatorial algorithm, construct the informational extended strategies $\mathbf{i}^{\alpha}=i_1^{\alpha}i_2^{\alpha}...i_j^{\alpha}...i_m^{\alpha}$ and $\mathbf{j}^{\beta}=j_1^{\beta}j_2^{\beta}...j_i^{\beta}...j_n^{\beta}$.

3. In parallel, all fixed MPI (α,β)-processes, using the OpenMP directives, construct utility matrices $A(\alpha,\beta)=||a_{i_j^{\alpha}j_i^{\beta}}||_{i\in I}^{j\in J}$ and $B(\alpha,\beta)=||b_{i_j^{\alpha}j_i^{\beta}}||_{i\in I}^{j\in J}$, generated by the informational extended strategies $\mathbf{i}^{\alpha}$ and $\mathbf{j}^{\beta}$.

4. In parallel, the α-rank MPI process, for all $\alpha=\overline{1,\varkappa_1}$, generates the "beliver-probabilities" $p(\beta/\alpha)$ for all fixed $\beta=\overline{1,\varkappa_2}$ of the α-type player 1 and also, β-rank MPI process, for all $\beta=\overline{1,\varkappa_2}$, generates the "beliver-probabilities" $q(\alpha/\beta)$ for all fixed $\alpha=\overline{1,\varkappa_1}$ of the β-type player 2.

5. Using MPI and OpenMP programming models, for all $\alpha=\overline{1,\varkappa_1}$, in parallel, the α-rank MPI process generates the sets $\mathbf{L}(\alpha)$ of the $\mathbf{l}^{\alpha}=l_1^{\alpha}l_2^{\alpha}...l_{\beta}^{\alpha}...l_{\varkappa_2}^{\alpha}$ strategies and constructs the payoff matrix $\mathbf{A}(\alpha)$ of the α-type player 1 and the β-rank MPI process, for all $\beta=\overline{1,\varkappa_2}$, generates the sets $\mathbf{C}(\beta)$ of the $\mathbf{c}^{\beta}=c_1^{\beta}c_2^{\beta}...c_{\alpha}^{\beta}...c_{\varkappa_1}^{\beta}$ strategies and constructs the payoff matrix $\mathbf{B}(\beta)$ of the β-type player 2. So all MPI (α,β)-processes have a pair of matrices $(\mathbf{A}(\alpha),\mathbf{B}(\beta))$.

7. In parallel, all MPI (α,β)-processes, using the OpenMP functions, ScaLAPACK routines and existing the sequential algorithm, determine all Nash equilibrium profiles in the bimatrix game with matrices $(\mathbf{A}(\alpha),\mathbf{B}(\beta))$.

8. Using ScaLAPACK-BLACS routines, the root MPI process gather from processes grid $\{(\alpha,\beta)\}_{\alpha=\overline{1,\varkappa_1}}^{\beta=\overline{1,\varkappa_2}}$ the sets of Nash equilibrium profiles in the bimatrix game $(\mathbf{A}(\alpha),\mathbf{B}(\beta))$.

In general case, to determine all sets of Bayes-Nash equilibrium profiles in bimatrix informational extended games a very large number (equal to $n^m \times m^n$) of the bimatrix subgames in the non extended strategies are to be solved.

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Limit theorems for stochastic equations involving local time of process

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It is well known that convergence of coefficients of Ito stochastic equation is not sufficient for weak convergence of solutions of stochastic equation. It is necessary additional condition.

We consider weak convergence of solutions of stochastic equations involving local time with nonregular dependence of the coefficients on small parameter ε:

$$\xi_\varepsilon(t) = x + \beta_\varepsilon L^{\xi_\varepsilon}(t,0) + \int_0^t \big(b_\varepsilon(\xi_\varepsilon(s)) + g_\varepsilon(\xi_\varepsilon(s))\big)ds + \\ + \int_0^t \sigma_\varepsilon(\xi_\varepsilon(s))dw(s). \qquad (1)$$

A stochastic equation involving local time was first investigated in [1] and [2]. Moreover in [2], [3], [4] were obtained formulae which connect solutions of stochastic equations with local time with solutions of Ito's stochastic equations.

We suppose that the coefficients of stochastic equation (1) satisfy following conditions: $\beta_\varepsilon \to \beta$ when $\varepsilon \to 0$, $|\beta_\varepsilon| < 1$ and $|\beta| < 1$, there exists a constant $\Lambda > 0$ such that $|g_\varepsilon(x)| < \Lambda$, $\frac{1}{\Lambda} < \sigma_\varepsilon^2(x) < \Lambda$ and for every $x \in \mathbb{R}$

$$\left| \int_0^x \frac{b_\varepsilon(y)}{\sigma_\varepsilon^2(y)} dy \right| \leq \Lambda \,.$$

Denote by $\xi(t)$ a weak solution following stochastic equation involving a local time (with $|\gamma| < 1$)

$$\xi(t) = x + \gamma L^\xi(t,0) + \int_0^t g(\xi(s))ds + \int_0^t \sigma(\xi(s))dw(s). \qquad (2)$$

Let $(\mathbb{C}[0,T], C_t), t \in [0,T]$ be a space of continuous functions on interval $[0,T]$. Let us denote as μ_ε and as μ the measures on functional space $(\mathbb{C}[0,T], C_t)$ generated by processes the $\xi_\varepsilon(t)$ and $\xi(t)$ respectively.

Theorem. *The necessary and sufficient conditions for the weak convergence of solutions of stochastic equations (1) to solution of stochastic equation (2) are obtained.*

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Perturbed homogeneous linear recurrent systems

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Let $a \in Rol[\mathbb{R}]$, $m = dim[\mathbb{R}](a)$ and $Q(z) = H_m^{[q]}(z) \in H[\mathbb{R}][m](a)$. From [2], a converges to L if and only if, for each root z_k of $Q(z)$, $|z_k| > 1$ or $z_k = 1$ is a simple root. If $Q(1) \neq 0$, then $L = 0$. Instead, if $Q(1) = 0$, we have $L \neq 0$ and L can be calculated, without knowing the roots of $Q(z)$, by transforming a into a $m-1$ linear recurrence with a constant inhomogeneity.

If $\exists z_k$ such that $|z_k| < 1$ or z_k is a multiple root with $|z_k| = 1$, then a diverges to infinity. Instead, if $\forall z_k$ is a simple root of unity, then a is periodic. When $\forall z_k$ is a simple root of unity or $|z_k| > 1$, then a is bounded.

The Jury Stability Criterion ([3]) can be applied for studying the localization of the roots of reciprocal polynomial $Q^*(z)$ in relation to unit circle, without finding the roots. We need to have at least $Q(1) > 0$, $Q(-1) > 0$ and $|q_{m-1}| < 1$ in order all the roots of $Q(z)$ to lie outside of unit disc. For instance, based on [1], this does not happen when $Q(z) \in \mathbb{Z}[z]$. Instead, the homogeneous linear recurrent distributions (including the distributions of stochastic systems with final sequence of states [2]) satisfy this property.

Next, we consider $u \in Rol[\mathbb{R}][m]$, $H_m^{[q]}(z) \in H[\mathbb{R}][m](u)$ and a small perturbation $\delta \in \mathbb{R}^m$ of q. We obtain the perturbed recurrence $v \in Rol[\mathbb{R}][m]$ with $I_m^{[v]} = I_m^{[u]}$ and $H_m^{[q+\delta]}(z) \in H[\mathbb{R}][m](v)$. The perturbation of u is $\varepsilon = v - u \in Rol[\mathbb{R}][2m]$ and $H(z) = H_m^{[q]}(z)H_m^{[q+\delta]}(z) \in H[\mathbb{R}][2m](\varepsilon)$.

The perturbation is stable if and only if $\varepsilon_n \to 0$ $(n \to \infty)$. From [2], $\varepsilon^2 \in Rol[\mathbb{R}][4m^2]$ and $s = \sum\limits_{n=0}^{\infty} \varepsilon_n^2 = G^{[\varepsilon^2]}(1)$. If ξ is a random variable with distribution $p = \varepsilon^2/s$, then its mode can be found using the successive search algorithm. The number of steps is limited by $n_r(\xi) = [\mathbb{E}(\xi) + \sigma(\xi)/\sqrt{p_r}]$, where r is the smallest integer for which $p_r > 0$ and the mode μ has probability $p_\mu = \max\limits_{r \le n \le n_r(\xi)} p_n$. This involves the estimation $|\varepsilon_n| \le \sqrt{sp_\mu}$, $\forall n \ge 0$.

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Average-discounted equilibria for stochastic positional games

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We study the problem of the existence and determining Nash equilibria for stochastic games with finite state and action spaces in the case when a part of players use limiting average reward payoffs criteria and an another part of players use the discounted reward payoffs criteria. Flesh, Thuijsman and Vrieze proved in [1] the existence of ϵ-equilibrium ($\forall \epsilon > 0$) in non-stationary strategies for two-playwer stochastic games, where player 1 uses the limiting average reward payoff ceiterion and player 2 usis the discounted reward payoff criterion. Such an equilibrium in [1] is called average-discounted equilibrium. Note, that for an average stochastic game, in general, a Nash equilibrium in stationary strategies may not exist [2], however for a discounted stochastic game a stationary Nash equilibrium always exists [3].

In this contribution we show that for an m-player stochastic game with finite state and action spaces, where one player uses the limiting average reward payoff criterion and the rest of players use the discounted reward payoffs criteria possesses an ϵ-equilibrium in non-stationary strategies. Additionally we show that for an arbitrary m-player stochastic positional game there exists a Nash equilibrium in stationary strategies. Based on constructive proof of this result we propose an algorithm for determining the optimal stationary strategies of the players.

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Modelling Performance Characteristics for Polling Models with semi-Markov Switching and Priorities

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Exhausting polling models with priorities and semi-Markov switching are considered. Some of performance characteristics, such as analog of Pollaczek-Khintchin virtual and steady state transform equations and analog of Gnedenko system's busy period are presented. Elaboration of numerical algorithms and modelling of performance characteristics are discussed.

Estimation of the exchange risk in conditions of the economic crisis, using statistical modeling

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The value of a currency depends on factors that affect the economy, such as trade, inflation, interest rates, growth rate, employment. Of course, without a good risk management plan as part of the process, a company will waste resources, time, and money, and will fail to manage their projects in the best way and correctly. It is very important to handle the risks for a good management of the company and we need to use new risk assessment and management techniques to track down potential and critical risks, and to develop the best strategies in this order.

There is a variety of models to calculate the Value-at-Risk, that are used in this order. The research for this problem is carried out by many authors in order to make an optimal management of the risks: [1]-[5], [7]. Among them, the more widely-used Statistical Methods to calculate VaR are: the historical simulation, the variance-covariance model, and Monte Carlo simulation. We can apply Monte Carlo simulation, which assumes that future currency returns will be randomly distributed [6], [8].

Some of models to calculate the VaR are applied and presented in the monograph [9]. One of the methods in [9] uses the game theory concept and is described as a dynamical game between the participants of a process of the currency exchange [10]. In this research paper we use some statistical methods in order to analyse the exchange rate fluctuations and to make some prediction, considering the high level of crisis and various factors that can affect a good dealing of projects inside the company. We use various statistical methods and introduce in the calculus the factor of inflation.

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Optimization methods for min-max fractional problems

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We consider the min-max fractional programming problem in which it is necessary to find

$$\nu^* = f(x^*) = \min_{x \in S} \max_{i \in I} \frac{\varphi_i(x)}{\psi_i(x)}, \tag{1}$$

where $\varphi_i(x), i \in I; -\psi_i(x), i \in I$ are convex functions in R^n and $\varphi_i(x) \geq 0, i \in I$. Additionally $\psi_i(x) > 0, i \in I$ for $x \in S$, where $S = \{x : h_k(x) \leq 0, k = \overline{1,p}\}$.

For studying and solving this problem we use the following two problems:

1. The generalized fractional-convex programming problem

$$\nu^* = f(x^*) = \min_{x \in S} \max_{i \in I} \frac{\sum\limits_{i \in I} u_i \varphi_i(x)}{\sum\limits_{i \in I} u_i \psi_i(x)},$$

where $U = \{u \in R^m : \sum_{i=1}^m u_i = 1; u_i \geq 0, i = \overline{1,m}\}$;

2. The parametric programming problem

$$F(u, \nu) = \min_{x \in S} \left[Z(x, u, \nu) = \sum_{i \in I} u_i(\varphi_i(x) - \nu \psi_i(x)) \right].$$

Using these auxiliary problems a special decomposition scheme and subgradient methods for solving problem (1) have been elaborated. Some approaches for studying and solving these class of problems are analysed in [1].

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The method for solving the multicriteria linear-fractional optimization problem in integers

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Numerous practical problems lead to solving multicriteria optimization problems. Of particular interest in this regard are the multicriteria optimization models in integers [1]. In the proposed paper we focused on solving the multicriteria optimization model in integers [2], in which the objective functions are of linear-fractional type. We studied the case where their denominators are identical, for which we bring an efficient solving algorithm. The algorithm is theoretically justified and has been tested on several examples. The mathematical model of the problem is the following:

$$\left\{ \begin{array}{c} \left\{ \begin{array}{c} \min \\ \max \\ x \in D \end{array} \right\} F_k\left(x\right) = \frac{\sum_{j=1}^{n} c_{kj} x_j}{\sum_{j=1}^{n} d_j x_j}, \quad k = \overline{1, r} \\ A \cdot x \leq b \\ x \in Z^{+} \end{array} \right. \tag{1}$$

in which: $D = \left\{ x = (x_1, x_2, \dots, x_n)^T \,|Ax \leq b, x \in Z^+ \right\}$, $A = \|a_{ij}\|$ is an array of size $m \times n (m < n)$, $C = \|c_{kj}\|$, is an array of size $r \times n (r < n)$, d is a n, -dimensional line vector. x is a vector n-dimensional column. and b is a m, -dimensional column vector.

A very important condition that we impose in order to solve the model (1) is that the function from the denominator, which is the same for all criteria, not to take the value zero on the domain D, i.e. the following relation is true:

$$\sum_{j=1}^{n} d_j x_j \neq 0, \quad (\forall)\, x \in D.$$

We note that in model (2) it is possible to have some criteria of minimum type and others of maximum type, for example, maximizing benefits, profit or others or minimizing costs, depreciation, loss or others.

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What is the chance to have a leader in a random set?

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Let us consider a population of individuals characterized by the same set of features. We denote the population by $(\mathbf{Z}_i)_{i\geq 1}$ and we assume that these are d-dimensional ($d \geq 2$) independent, identically distributed random vectors. Let us select a finite sample: $\mathcal{S}_n = \{\mathbf{Z}_1, \mathbf{Z}_2, .., \mathbf{Z}_n\}$, $n \geq 1$.

If there exists $\mathbf{Z}_j \in \mathcal{S}_n$ such that $\mathbf{Z}_j \geq \mathbf{Z}_k$, $\forall 1 \leq k \leq n$, then we say that $\mathbf{Z}_j$ is a leader of $\mathcal{S}_n$.

If there exists $\mathbf{Z}_j \in \mathcal{S}_n$ such that $\mathbf{Z}_j \leq \mathbf{Z}_k$, $\forall 1 \leq k \leq n$, then we dename $\mathbf{Z}_j$ an anti-leader of $\mathcal{S}_n$.Here the comparison of two vectors has the usual sense: if $\mathbf{Z}_j = \left(Z_1^{(j)}, Z_2^{(j)}, ..., Z_d^{(j)}\right)$ and $\mathbf{Z}_k = \left(Z_1^{(k)}, Z_2^{(k)}, ..., Z_d^{(k)}\right)$ then $\mathbf{Z}_j \geq \mathbf{Z}_k \Leftrightarrow Z_i^{(j)} \geq Z_i^{(k)}$, $\forall 1 \leq i \leq d$.

Our purpose is to compute (if possible) or to estimate the probability that a leader (or an anti-leader, or both) does exist in a given sample.

We focus our study on a particular case: precisely we consider $\mathbf{Z} = f(X)$ with $f = (f_1, f_2, ..., f_d) : [0, 1] \to \mathbb{R}^d$, X a random variable uniformly distributed on $[0, 1]$ and, in most examples, $f_1(X) = X$.

6. Algebra, Logic, Geometry, Topology

Exact modules over CDGA and weight systems

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We introduce a general method for the construction of weight systems. Our construction gives a conceptual framework for the weight systems existing in the literature, including the hyperkahler ones and those coming from Lie algebroids.

Properties of coverings in lattices of ring topologies

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This work is a continuation of studies that were in the work [1].

Definition. *Let $(\Delta, <)$ be a lattice and let $a, b \in \Delta$. If $a < b$ and between elements a and b there exist no other elements in the lattice Δ then we shall say that the element b covers the element a in the lattice Δ.*

We shall denote this by $a \prec_\Delta b$.

Remark 1. If I is an ideal of a ring R then the set $\{I\}$ is a basis of filter of neighborhoods of zero for a ring topology.

We shall denote this topology by $\tau(I)$.

Theorem *Let Q be a ring and let Δ be the lattice of all ring topologies on the ring Q or the lattice of all ring topologies of the ring Q in each of which the topological ring has a basis of filter of neighborhoods of zero which consists of subgroups of the additive group of the ring Q. If τ_1 and τ_2 are ring topologies such that $\tau_1 \prec_\Delta \tau_2$ and $\{U_\gamma | \gamma \in \Gamma\}$ and $\{V_\beta | \beta \in \mathrm{B}\}$ are bases of filters of neighborhoods of zero in topological rings (Q, τ_1) and (Q, τ_2), respectively, then for any ideal I of ring Q the following statements are true*:

1. *There exist ring topologies $\bar{\tau}_1$ and $\bar{\tau}_2$ such that families $\{I + U_\gamma | \gamma \in \Gamma\}$ and $\{I + V_\beta | \beta \in \mathrm{B}\}$ are bases of filters of neighborhoods of zero in the topological rings $(Q, \bar{\tau}_1)$ and $(Q, \bar{\tau}_2)$ respectively and either $\bar{\tau}_1 \prec_\Delta \bar{\tau}_2$ or $\bar{\tau}_1 = \bar{\tau}_2$.*

2. *Either* $\sup\{\tau(I), \tau_1\} = \sup\{\tau(I), \tau_2\}$ *or the set $\{(U_\gamma \bigcap I) + V_\beta | \gamma \in \Gamma, \beta \in \mathrm{B}\}$ will be a basis of filter of neighborhoods of zero in the topological ring (Q, τ_1).*

3 *If* I *is an ideal of the ring Q such that sup*$\{\tau_1, \tau(\mathrm{I})\} \neq \sup\{\tau_2, \tau(\mathrm{I})\}$ *and* I *is an open ideal in the topological ring (Q, τ_2), then* I *is an open ideal in the topological ring* (Q, τ_1) too.

Remark 2. A ring R, an ideal I and two ring topologies τ_1 and τ_2 in each of which the topological ring has a basis of filter of neighborhoods of zero which consists of subgroups of the additive group of the ring are constructed such that $\tau_1 \prec_\Delta \tau_2$ and between topologies $sup\{\tau(I), \tau_1\}$ and $sup\{\tau(I), \tau_2\}$ there are infinite number of ring topologies in each of which the topological ring has a basis of filter of neighborhoods of zero which consists of subgroups of the additive group of the ring.

This example shows that the given in [1] conditions under which the properties of a unrefinable chain of ring topologies, are preserved under taking the supremum are essential.

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Properties of Boroczky's construction

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Of a special interest are tilings in hyperbolic n–space Λ^n. It is natural to extend the study of tiling problems to the hyperbolic plane as well as hyperbolic spaces of higher dimension. In 1974, K. Böröczky published a construction of tilings of hyperbolic plane Λ^2 by a single prototile. In plain words, the construction goes as follows. Let us describe it for 3-space Λ^3 first. Consider a collection of concentric horospheres, where consecutive horospheres have equal distance. Each horosphere is conformal to the Euclidean plane $\mathbb{R}^2$. So consider a partition of each horosphere into the canonical tiling of $\mathbb{R}^2$ by unit squares. Erect on each square a prism, such that the top of the prism is made of four squares of the next layer. This yields a tiling of Λ^3, where each tile carries four tiles on its top. These "polyhedral layers" fit together and produce the Böröczky tiling of the whole hyperbolic 3-space. This construction can be extended to any dimension, yielding tilings of hyperbolic n–space Λ^n.

To obtain corresponding non face-to-face tiling of 3-space Λ^3 into convex prismatic equal hexa-faceted polyhedra it is enough every nine-faceted polyhedra of Böröczky's tiling to cut into four prismatic polyhedra its coordinate planes of symmetry. The tilings (face-to-face and non-face-to-face) of n–dimensional hyperbolic space Λ^n are under construction almost literally in the same way through partition of corresponding $(n-1)$–horospheres into geodesic $(n-1)$–cube (cubiliaj).

Theorem 1. *In the hyperbolic 3–space Λ^3, there exists a non-regular non face-to-face tiling (non-normal) composed of congruent convex polyhedral tiles, which can't be transformed into regular tiling using any permutation of the polyhedral tiles.*

Our proof makes use of the so-called Böröczky tiling of hyperbolic Λ^3 by congruent polyhedra.

An analogous construction works for arbitrary dimension.

Theorem 2. *In the hyperbolic n–space Λ^n, there exists a non-regular non face-to-face (non-normal) tiling composed of congruent convex polyhedral tiles, which can't be transformed into regular tiling using any permutation of the polyhedral tiles.*

The proposed construction can be considered and as the constructive proof of the theorem of the existence of non-face-to-face tilings in the n – dimensional hyperbolic space into equal, convex and compact polyhedra. The work outlined some possible generalizations of Boroczky's construction, which in most cases, also allow to construct and non-face-to-face tilings. Features of tilings can constructively prove some general statements concerning, for example, point Delone Sets and Delone tilings. In the article it is also discussed the question of the number of hyperfaces for hyperbolic n–dimensional tile.

The question-behavior of geodesics on hyperbolic manifolds: analysis

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We are concerned in this paper with the behavior in the large of the geodesic lines on hyperbolic manifolds, giving special atention to the two-dimensional cases. A geodesic in a hyperbolic surfaces is an arc which in the local coordinate charts, is the image of a geodesic arc of the hyperbolic plane. Topological surfaces are often thought of as the result of pasting togetheter polygons. Provided you have enough topology, pants decompositions are a natural way of decomposing (orientable) surfaces or conversely one can build a surface by pasting 3 holed spheres (pants) a long their cufs. So it is important to study the behavior of geodesics on a pair of pants. Thanks to the development of the new constructive approach, in this paper, the author succeeded to receive "in a certain sense" the solution for the behavior of the geodesics in general on the hyperbolic manifolds, structure of geodesics and their types. Arbitrary hyperbolic surfaces M, closed or open, of finite or infinite genus are considered. Yet another way to define a hyperbolic surface is via its universal cover. For the behavior of the geodesics on the specified fragments (hyperbolic pants, etc.) it is used a certain figure, named in the text of the work the multilateral. The study of the behavior of the geodesics in this paper is being carried out gradually, in order of collecting the surface, the reverse order of cutting the surface into fragments (i.e. pants). The surface is cut into typical pieces (for example, on pants or their degenerations, on right hexagons, etc.) and the question of the behavior of the geodesics for each piece is solved on it, and then the result of the investigation returns (by gluing) onto the original surface. With the help of these multilaterals, it is possible to determine the nature of the behavior of the geodesics on the surface. Any given hyperbolic (closed, i.e., ordinary) surface can be cut into pants and the question is how, when gluing such pants, connect them on a common surface. But it may seem (when gluing of the surface from the pants is not finished yet) that the surface of genus g has also n components (the surface has a geodesic boundary). And, going further, we notice that the boundary of the surface can degenerate: transform into cuspidal ends (cusps) and into conical points. Thus, we arrive at the most general case, the surfaces of the signature (g, n, k), the preliminary investigation of the behavior of the geodesics on these pieces. A concrete method of investigating the behavior of the geodesics on hyperbolic 2-manifolds is based on the idea of preliminary research on these pieces (on the set of hyperbolic pants and their degenerations), in the subsequent consolidation of research results using the method proposed in this paper (sometimes called the method of generalized coloured multilaterals). The solution is based on the study of the behavior of the geodesics on the simplest hyperbolic surfaces (hyperbolic pants, degenerate hyperbolic pants, thrice-punctured sphere, etc.), some of which have long attracted the attention of geometers.

On minimal Endo hypersurfaces in nearly Kählerian manifolds

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As it is known, one of the most important examples of almost contact metric structures is the structure induced on an oriented hypersurface in an almost Hermitian manifold [1], [2]. We recall that an almost contact metric structure on a manifold N of odd dimension is defined by a system of tensor fields $\{\Phi,\ \xi,\ \eta,\ g\}$ on this manifold: here ξ is a vector, η is a covector, Φ is a tensor of the type (1, 1) and $g = \langle\cdot,\cdot\rangle$ is the Riemannian metric. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1,\ \Phi(\xi) = 0,\ \eta\circ\Phi = 0,\ \Phi^2 = -id + \xi\otimes\eta,$$
$$\langle\Phi X, \Phi Y\rangle = \langle X, Y\rangle - \eta\,(X)\,\eta\,(Y)\,,\ X, Y \in \aleph(N),$$

where $\aleph(N)$ is the module of C^{∞}-smooth vector fields on the manifold N [1]. The almost contact metric structure is called Endo (or nearly cosymplectic), if

$$(\nabla_X\,\Phi)\ X\ =\ 0,\ X \in \aleph(N).$$

In the present note, Endo hypersurfaces in nearly Kählerian manifolds (NK-manifolds, or W_1-manifolds, using Gray–Hervella notation [3]) are considered. Such Endo hypersurfaces are characterized in terms of their type number. A simple criterion of the minimality of Endo hypersurfaces of nearly Kählerian manifolds is established. The following Theorem contains the main result of our note.

Theorem. *If N is an Endo hypersurface in a nearly Kählerian manifold M^{2n} and t is its type number, then the following statements are equivalent:*
1) N is a minimal hypersurface in M^{2n};
2) N is a totally geodesic hypersurface in M^{2n};
3) $t \equiv 0$.

Taking into account that the six-dimensional sphere is an example of a nearly Kählerian manifold [1], [4], we deduce that there are not minimal non-geodesic Endo hypersurfaces in S^6 .

We remark that this work is a continuation of the researches of the author, who studied diverse almost contact metric structures on oriented hypersurfaces in nearly Kählerian manifolds [5]–[9].

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Hermitian geometry of six-dimensional planar submanifolds of Cayley algebra

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1. The almost Hermitian structures belong to the most important and meaningful differential-geometrical structures. The existence of Gray-Brown 3-vector cross products [1] in Cayley algebra gives a set of substantive examples of almost Hermitian manifolds. As it is well known, every 3-vector cross product on Cayley algebra induces a 1-vector cross product (or, what is the same in this case, an almost Hermitian structure) on its six-dimensional oriented submanifold [2], [3]. Such almost Hermitian structures (in particular, Kählerian, nearly Kählerian, quasi Kählerian, Hermitian, special Hermitian etc) were studied by a number of outstanding geometers: E. Calabi, A. Gray, V. F. Kirichenko, K. Sekigawa and L. Vranchen.

We recall that an almost Hermitian manifold is a $2n$-dimensional manifold M^{2n} with a Riemannian metric $g = \langle \cdot, \cdot \rangle$ and an almost complex structure J. Moreover, the following condition must hold

$$\langle JX,\ JY \rangle = \langle X,\ Y \rangle\,, \quad X, Y \in \aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on M^{2n}. An almost Hermitian manifold is Hermitian, if its almost complex structure is integrable [4].

2. In the present work, we consider six-dimensional Hermitian planar submanifolds of Cayley algebra. We present the following results.

Theorem 1. *If a six-dimensional Hermitian planar submanifold of Cayley algebra satisfies the U-Kenmotsu hypersurfaces axiom, then it is Kählerian.*

Theorem 2. *A symmetric non-Kählerian Hermitian six-dimensional submanifold of Ricci type does not admit totally umbilical Kenmotsu hypersurfaces.*

Theorem 3. *If a six-dimensional Hermitian planar submanifold of general type of Cayley algebra satisfies the 1-cosymplectic hypersurfaces axiom, then it is Kählerian.*

Theorem 4. *The Hermitian structure on a 6-dimensional planar submanifold of Cayley algebra is stable if and only if such submanifold is totally geodesic.*

Theorem 5. *The quasi-Sasakian structure on a totally umbilical hypersurface of a six-dimensional Hermitian planar submanifold of Cayley algebra is Sasakian.*

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Weak reflexive subcategory

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In the subcategory of topological vector locally convex spaces Hausdorff are built proper classes of weakly reflective and weakly coreflective subcategories, respectively. We proved that the right product of two subcategories often leads us to a weakly reflective subcategory, which is not reflective. The problem formulated in the paper is answered [2], when the right product is a reflective subcategory. For notations, see [1-2].

Definition. *A full subcategory $\mathcal{R}$ of category $\mathcal{C}$ is called weakly reflective, if for any $X \in |\mathcal{C}|$ there is an object $rX \in |\mathcal{R}|$ and a morphism $r^X : X \to rX$ with the property: for an object $A \in |\mathcal{R}|$ any morphism $f : X \to A$ extends through r^X : $f = g \cdot r^X$ for un g. If the extension is always unique, then $\mathcal{R}$ is called the reflective subcategory.*

We denote by $\mathbb{R}$ (respectively $\mathbb{K}$) the class of non-zero subcategories of category $\mathcal{C}_2\mathcal{V}$. For $\mathcal{K} \in \mathbb{K}$ and $\mathcal{R} \in \mathbb{R}$ with the respective functors $k : \mathcal{C}_2\mathcal{V} \to \mathcal{K}$ and $r : \mathcal{C}_2\mathcal{V} \to \mathcal{R}$, either $\mu\mathcal{K} = \{m \in \mathcal{M}ono | k(m) \in \mathcal{I}so\}$, $\varepsilon\mathcal{R} = \{e \in \mathcal{E}pi | r(e) \in \mathcal{I}so\}$. Both $\mu\mathcal{K}$ and $\varepsilon\mathcal{R}$ are classes of bimorphisms.

Definition. 1. *Let $\mathcal{K} \in \mathbb{K}$ and $\mathcal{R} \in \mathbb{R}$ be. The subcategory $\mathcal{S}_{\mu\mathcal{K}}(\mathcal{R})$ is called the right product of subcategories $\mathcal{K}$ and $\mathcal{R}$ and it is noted by $\mathcal{K} *_d \mathcal{R}$.*

2. *The subcategory $\mathcal{Q}_{\varepsilon\mathcal{R}}(\mathcal{K})$ is called the left product of subcategories $\mathcal{K}$ and $\mathcal{R}$ and it is noted by $\mathcal{K} *_s \mathcal{R}$.*

If $\mathcal{B}$ is a class of bimorphisms and $\mathcal{A}$ a subcategory, then $S_{\mathcal{B}}(\mathcal{A})$ (respectively $Q_{\mathcal{B}}(\mathcal{A})$) is the full subcategory of all $\mathcal{B}$-subobjects (respectively: $\mathcal{B}$-factorobjects) of the objects of subcategory $\mathcal{A}$.

We examine the following two conditions: a) $\mathcal{R}$ contains the subcategory $\mathcal{S}$ of spaces with weak topology; b) $\mathcal{K}$ contains the subcategory $\tilde{\mathcal{M}}$ of spaces with Mackey topology.

Theorem 1. *The equality $r^X = u^X \cdot v^X$ is the $((\mu\mathcal{K})^\top, \mu\mathcal{K})$-factorization of morphism r^X.*

1^*. *The equality is the $((\varepsilon\mathcal{R}), \varepsilon\mathcal{R}^\perp)$-factorization of morphism k^X.*

2. *$S_{\mu\mathcal{K}}(\mathcal{R})$ is a weak reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$ and $v^X : X \to vX$ is the weak replique of object X.*

2^*. *$Q_{\varepsilon\mathcal{R}}(\mathcal{K})$ is a weak coreflective subcategory of the category $\mathcal{C}_2\mathcal{V}$ and $w^X : wX \to X$ is the weak coreplique of object X.*

3. *$S_{\mu\mathcal{K}}(\mathcal{R})$ is a reflective subcategory if it meets one of the conditions a) or b).*

3^*. *$Q_{\varepsilon\mathcal{R}}(\mathcal{K})$ is a coreflective subcategory if it meets one of conditions a) or b).*

Theorem 2. *Let $\mathcal{R} \in \mathbb{R}$ be, Σ the coreflective subcategory of the spaces with the most powerful locally convex topology and $\sigma : \mathcal{C}_2\mathcal{V} \to \Sigma$. The following statements are equivalent:*

1. *$\mathcal{S}_{\mu\Sigma}(\mathcal{R})$ is a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$.*

2. $\mathcal{S} \subset \mathcal{R}$.

Theorem 3. *Let $\mathcal{K} \in \mathbb{K}$ be, Π the reflective subcategory of the complete spaces with weak topology and $\pi : \mathcal{C}_2\mathcal{V} \to \Pi$. The following statements are equivalent:*

1. $\mathcal{Q}_{\varepsilon\Pi}(\mathcal{K})$ *is a coreflective subcategory of the category* $\mathcal{C}_2\mathcal{V}$.

2. $\tilde{\mathcal{M}} \subset \mathcal{K}$.

Theorem 4. 1. *$\mathcal{K} *_d \mathcal{R}$ is a reflective subcategory iff when there is a coreflective subcategory $\mathcal{K}_0$ so that $\widetilde{\mathcal{M}} \subset \mathcal{K}_0$ and $\mathcal{K} *_d \mathbb{R} = \mathcal{K}_0 *_d \mathbb{R}$.*

2. *$\mathcal{K} *_s \mathbb{R}$ is a coreflective subcategory iff when there is a coreflective subcategory $\mathcal{R}_0$ so that $\mathcal{S} \subset \mathcal{R}_0$ and $\mathcal{K} *_s \mathbb{R} = \mathcal{K} *_s \mathbb{R}_0$.*

Examples 1. Let $\sum *_d \mathcal{R}$ be a reflective subcategory. Then $\mathcal{S} \subset \mathcal{R}$, $\sum_0 = Q_{\mathcal{E}_p(\sum)} = \tilde{\mathcal{M}}$ and $\sum *_d \mathcal{R} = \tilde{\mathcal{M}} *_d \mathcal{R} = \mathcal{C}_2\mathcal{V}$.

1^*. Let $\mathcal{K} *_s \prod$ be a coreflective subcategory. Then $\tilde{\mathcal{M}} \subset \mathcal{K}$, $\mathcal{S}_{\mathcal{M}_p(\prod)} = \mathcal{S}$ and $\mathcal{K} *_s \prod = \mathcal{K} *_s \mathcal{S} = \mathcal{C}_2\mathcal{V}$.

2. Let $\mathcal{K} \in \mathbb{K}$, $\mathcal{R} \in \mathbb{R}$ and $\mathcal{K} \subset \tilde{\mathcal{M}}$ be. Then $\mathcal{K} *_d \mathcal{R} = \tilde{\mathcal{M}} *_d \mathcal{R}$.

2^*. Let $\mathcal{K} \in \mathbb{K}$, $\mathcal{R} \in \mathbb{R}$ and $\mathcal{R} \subset \mathcal{S}$ be. Then $\mathcal{K} *_s \mathcal{R} = \mathcal{K} *_s \mathcal{S}$.

3. Let $\mathcal{K} \in \mathbb{K}$, $\mathcal{K}_0 = Q_{\mathcal{E}_p}(\mathcal{K})$, $\mathcal{T} \in \mathbb{K}$ and $\mathcal{K} \subset \mathcal{T} \subset \mathcal{K}_0$ be. Then $\mathcal{T} *_d \mathcal{R} = \mathcal{K}_0 *_d \mathcal{R}$.

3^*. Let $\mathcal{R} \in \mathbb{R}$, $\mathcal{R}_0 = S_{\mathcal{M}_p}(\mathcal{R})$, $\mathcal{U} \in \mathcal{K}$ and $\mathcal{R} \subset \mathcal{U} \subset \mathcal{R}_0$ be. Then $\mathcal{K} *_s \mathcal{U} = \mathcal{K} *_s \mathcal{R}_0$.

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The groupoid of c-reflective subcategories

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the category $\mathcal{C}_2\mathcal{V}$, of topological vector locally convex Hausdorff spaces in the class of c-reflective subcategories $\mathbb{R}_c$, a binary operation is introduced so that $\mathbb{R}_c$ becomes a groupoid (commutative, with neutral element, any element is its symmetric).

$\mathbb{R}_c$ is the class of reflective subcategories $\mathcal{L}$ that contain the subcategory of spaces with weak topology $\mathcal{S}$ and the reflective functor $l : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{L}$ is exactly on the left. Such subcategories are $\mathcal{S}$, the subcategory of ultranuclear spaces, Schwartz spaces, etc. (see [2]).

Let $\mathcal{L}$, $\mathcal{R} \in \mathbb{R}$ be and $\rho(\mathcal{L},\mathcal{R})$ the full subcategory of the category $\mathcal{C}_2\mathcal{V}$, of those objects A, for which the $\mathcal{L}$-replica and $\mathcal{R}$-replica are isomorphic: $lX = rX$. For $\mathcal{L} \in \mathbb{R}$ let $\varepsilon\mathcal{L} = \{e \in \mathcal{E}pi | l(e) \in \mathbb{I}so\}$.

Theorem *Let $\mathcal{L}$, $\mathcal{R} \in \mathbb{R}_c$ and $\mathcal{B} = (\varepsilon\mathcal{L}) \cap (\varepsilon\mathcal{R})$ be. Then:*

1. *$\rho(\mathcal{L},\mathcal{R})$ is a reflective subcategory and $\mathcal{S} \subset \rho(\mathcal{L},\mathcal{R})$.*

2. *$\rho(\mathcal{L},\mathcal{R})$ is closed in relation to $\mathcal{B}$-subobjects and $\mathcal{B}$-factorobjects: $\rho(\mathcal{L},\mathcal{R})$ is a $\mathcal{B}$-semireflexive subcategory [1].*

Any reflective subcategory that contains subcategory $\mathcal{S}$ also contains the largest c-reflective subcategory. For $\rho(\mathcal{L},\mathcal{R})$ we note it $\overline{\rho}(\mathcal{L},\mathcal{R})$.

The binary operation $\mathcal{L} \oplus \mathcal{R} = \overline{\rho}(\mathcal{L},\mathcal{R})$ in the class $\mathbb{R}_c$ possesses the following properties:

1. It is a commutative operation: $\mathcal{L} \oplus \mathcal{R} = \mathcal{R} \oplus \mathcal{L}$.

2. The element $\mathcal{C}_2\mathcal{V} \in \mathbb{R}_c$ is a neutral element:

$$\mathcal{L} \oplus \mathcal{C}_2\mathcal{V} = \mathcal{L}.$$

3. Any element $\mathcal{L} \in \mathbb{R}_c$ is also its symmetrical:

$$\mathcal{L} \oplus \mathcal{L} = \mathcal{C}_2\mathcal{V}.$$

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Generalized Hausdorff compactifications

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A generalized extension or a g-extension of a space X is a pair (Y, f), where $f : X \to Y$ is a continuous mapping and the set $f(X)$ is dense in Y. If f is an embedding, then Y is an extension of X and it is assumed that $X = f(X)$ and $f(x) = x$ for any $x \in X$. Let $GE(X)$ be the set of all g-extension of the space X and $E(X)$ be the set of all extensions of X. Obviously, $E(X) \subseteq GE(X)$. If P is a topological property, then $PGE(X)$ is the set of all g-extensions with the property P and $PE(X) = E(X) \cap PGE(X)$.

A set $L \subseteq PGE(X)$ is a complete lattice of g-extensions of X if for every non-empty set $H \subseteq L$ we have $(\vee H) \cap L \neq \emptyset$ and $(\wedge H) \cap L \neq \emptyset$.

Let X be a T_0-space. Let us denote by $HGC(X)$ the set of the g-compactifications (bX, b_X) of the space X for which bX is a Hausdorff space.

Theorem 1. *The set of $HGC(X)$ is a complete lattice of g-extensions with the maximal element.*

The maximal element of the lattice $HGC(X)$ is denoted by $(\beta X, \beta_X)$ and is called g-compactification Stone-Čech.

Corollary 1. *If $f : X \to Y$ is a continuous application, then there is a single continuous application $\beta f : \beta X \to \beta Y$ for which $\beta f \circ \beta_X = \beta_Y \circ f$.*

Corollary 2. *If $f : X \to Y$ is a continuous application of the space X in the Hausdorff space and compact Y, then there is a single continuous application $\beta f : \beta X \to Y$ for which $f = \beta f \circ \beta_X$.*

Theorem 2. ([2], for T_1-spaces). *For any continuous application of space X in a Hausdorff and compact space Y, there is a single continuous application $\omega f : \omega X \to Y$ for which $f = \omega X|X$.*

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Preradicals and closure operators in modules: comparative analysis and relations

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The theory of radicals in modules is based by the notion of *preradical* (as subfunctor of identical functor) [1]. The other important notion of the modern algebra is the *closure operator* (as a function C which by every pair of modules $N \subseteq M$ defines a submodule $C_M(N) \subseteq M$, C being compatible by the morphisms of R-Mod) [2]. The purpose of this communication consists in the elucidation of the relations between these fundamental notions and the comparison of results of those respective theories. The closure operators in some sense are the generalization of preradicals, since the class $\mathbb{PR}(R)$ can be inserted in $\mathbb{CO}(R)$ (by two methods). This important fact determines a close connection between the results of the respective domains.

In particular, there exists some correspondences between the main types of preradicals of R-Mod and respective types of closure operators. Also there exists a remarkable connection between the operations in the big lattices $\mathbb{PR}(R)$ and $\mathbb{CO}(R)$. These facts show the parallelism and similarity of two theories. However, it is obvious that $\mathbb{CO}(R)$ is essentially ,,larger" than $\mathbb{PR}(R)$ (closure operators are the functions of two variables). Therefore in the study of closure operators we must apply both the classical methods of radical theory, adding the constructions, notions and applications, related by the specificity of closure operators.

In continuation we formulate some typical results of this domain.

Theorem 1. *There exists a monotone bijection between:*
a) the maximal closure operators of R-Mod and the preradicals of R-Mod: $Max[\mathbb{CO}(R)] \cong \mathbb{PR}(R)$;
b) the minimal closure operators of R-Mod and the preradicals of R-Mod: $Min[\mathbb{CO}(R)] \cong \mathbb{PR}(R)$;
b) the equivalence classes of $\mathbb{CO}(R)$*, determined by the relation ,,*$\sim$*"* $(C \sim D \overset{def}{\Longleftrightarrow} r_C = r_D)$*, and the preradicals of R-Mod:* $\mathbb{CO}(R)/_{\sim} \cong \mathbb{PR}(R)$.

Theorem 2. *There exists a monotone bijection between:*
a) the idempotent preradicals of R-Mod and the closure operators

which are maximal and weakly hereditary;
b) the radicals of R-Mod and the closure operators which are maximal and idempotent;
c) the pretorsions (torsions) of R-Mod and the closure operators which are minimal and hereditary (maximal, idempotent and hereditary).
Theorem 3. *There exists a monotone bijection between the cohereditary closure operators of R-Mod and the ideals of R.*

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Groupoids of order three up to isomorphisms

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List of all classical Bol-Moufang identities is given in [1]. We continue count of number non-isomorphic groupoids of order three with some Bol-Moufang identities [2, 3, 4, 5].

There exist 26 non-isomorphic groupoids of order 3 with identity F_9, (x.yz)x= x(yz.x) from possible 221 groupoids.

There exist 159 non-isomorphic groupoids of order 3 with identity F_{10}, x(y. zx) = x(yz.x) from possible 874 groupoids.

Identity F_{11} xy.xz = (xy,x)z: up to isomorphism there exist 49 groupoids.

Identity F_{12} xy.xz = (x.yx)z: up to isomorphism there exist 45 groupoids.

Identity F_{13} xy.xz = x(yx.z): up to isomorphism there exist 53 groupoids.

Identity F_{13} xy.xz = x(yx.z): up to isomorphism there exist 53 groupoids.

Identity F_{14} xy.xz = x(y.xz): up to isomorphism there exist 61 groupoids.

Identity F_{15} (xy.x)z = (x.yx)z: up to isomorphism there exist 253 groupoids.

Identity F_{16} (xy.x)z = x(yx.z): up to isomorphism there exist 73 groupoids.

Identity F_{17} (xy.x)z = x(yx.z): up to isomorphism there exist 35 groupoids.

Identity F_{18} (x.yx)z = x(yx.z): up to isomorphism there exist 61 groupoids.

Identity F_{19} (x.yx)z = x(y.xz): up to isomorphism there exist 40 groupoids.

Identity F_{20} x(yx.z) = x(y.xz): up to isomorphism there exist 110 groupoids from possible 601.

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On some non-isomorphic quasigroups of small order

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A non-empty set G is said to be a *groupoid* relatively to a binary operation denoted by $\{\cdot\}$, if for every ordered pair (a, b) of elements of G there is a unique element $ab \in G$.

A groupoid $(G, \cdot)$ is called a *quasigroup* if for every $a, b \in G$ the equations $a \cdot x = b$ and $y \cdot a = b$ have unique solutions.

We consider the quasigroups of properties $(*)$: left distributive, right distributive, left Bol, right Bol, left involutary, right involutary, left semimedial, right semimedial, left cancellative, right cancellative, non-associative loops, medial, paramedial, Ward, Cote and Manin quasigroups. We examine the following problem:

Problem 1. How many non-isomorphic quasigroups of properties $(*)$ of order 3, 4, 5, 6, 7 do there exist?

We have elaborated algorithms for generating and enumerating non-isomorphic quasigroups of small order of properties $(*)$. The results established here are related to the work in ([1,2,3,4,5]).

Applying the algorithms elaborated, we prove the following results:

The number of some non-isomorphic quasigroups						
Nr	**Property of quasigroup**	**The order of quasigroups/ number**				
		3	**4**	**5**	**6**	**7**
1	the number of non-isomorphic quasigroups	5	35	1411	1130531	-
2	commutative quasigroup $xy = yx$	3	7	11	491	6381
3	the number of non-isomorphic groups	1	2	1	2	1
4	idempotent quasigroup $xx{=}x$	1	1	4	18	10 213
5	left distributive quasigroup $x{\cdot}(yz){=}(xy)\cdot(xz)$	1	1	3	0	5
6	right distributive quasigroup $(xy)\cdot z = (xz)\cdot(yz)$	1	1	3	0	5
7	left Bol quasigroup $y(z(yx)) = (y(zy)x$	2	4	2	4	2
8	right Bol quasigroup $((xy)z)y{=} x((yz)y)$	2	4	2	4	2
9	leftinvolutary $x \cdot xy = x$	3	7	11	491	6381

The number of some non-isomorphic quasigroups						
Nr	**Property of quasigroup**	**The order of quasigroups/number**				
		3	**4**	**5**	**6**	**7**
10	rightinvolutary $xy \cdot y = x$	3	7	11	491	6381
11	left semimedial quasigroup $xx \cdot yz = xy \cdot xz$	5	13	19	8	41
12	right semimedial quasigroup $zy \cdot xx = zx \cdot yz$	5	13	19	8	41
13	left cancellative quasigroup $xy = xz \Longrightarrow y = z$	5	35	1411	1130531	-
14	right cancellative quasigroup $yx = zx \Longrightarrow y = z$	5	35	1411	1130531	-
15	loop	1	2	6	109	23746
16	medial quaisgroup $(xy) \cdot (uv)= (xu) \cdot (yv)$	5	13	19	5	41
17	paramedial quasigroup $(xy) \cdot (uv) = (vy) \cdot (ux)$	5	11	9	5	13
18	medial and paramedial quasigroup	5	9	9	5	13
19	Ward quasigroup $(ac)(bc) = ab$	1	2	1	2	-
20	Cote quasigroup $a(ab \cdot c) = (c \cdot aa)b$	3	4	2	3	-
21	Manin quasigroup $a(b \cdot ac) = (aa \cdot b)c$	3	4	4	3	-

Table 1. **The number and properties of non-isomorphic quasigroups of small order**

Proposition. *The number of non-isomorphic groups of order* $n = 1, 2, \ldots, 90$ *is determined by the following string of numbers in* *the given order: 1, 1, 1, 2, 1, 2, 1, 5, 2, 2, 1, 5, 1, 2, 1, 14, 1, 5, 1, 5, 2, 2, 1, 15, 2, 2, 5, 4, 1, 4, 1, 51, 1, 2, 1, 14, 1, 2, 2, 14, 1, 6, 1, 4, 2, 2, 1, 52, 2, 5, 1, 5, 1, 15, 2, 13, 2, 2, 1, 13, 1, 2, 4, 267, 1, 4, 1, 5, 1, 4, 1, 50, 1, 2, 3, 4, 1, 6, 1, 52, 15, 2, 1, 15, 1, 2, 1, 12, 1, 10.*

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On Topological CM-quasigroups

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The authors introduced the notion of *CM-groupoid* to the memory of Professor Choban Mitrofan who founded the school of topological algebra at the Tiraspol State University [1].

A groupoid $(G,\cdot)$ is called *CM-groupoid* if it satisfies the law $xy \cdot z = y \cdot zx$ for all $x, y, z, t \in G$. A groupoid $(G,\cdot)$ is called a *quasigroup* if for every $a, b \in G$ the equations $a \cdot x = b$ and $y \cdot a = b$ have unique solutions.

The notion of multiple identities was introduced in [2].

Theorem 1. *If $(G,\cdot)$ is a CM-multiplicative groupoid, $e \in G$ and the following conditions hold:*

1. $ex = x$ for every $x \in G$,

2. $x^2 = x \cdot x = e$ for every $x \in G$,

3. if $xa = ya$ then $x = y$ for all $x, y, a \in G$,

then $(G,\cdot)$ is a CM-quasigroup with an $(1,2)$-identity e.

Corolary. *If $(G,\cdot)$ is a CM-quasigroup with an $(1,2)$-identity e and $x^2 = e$, then solutions of the equations $ax = b$ and $ya = b$ are respectively $x = ab$ and $y = a \cdot be$, for all $x, y, a, b \in G$.*

Proposition 1. *Let $(G,\cdot)$ be a topological CM-quasigroup $(G,\cdot)$ with an $(1,2)$-identity e. Then the mapping $f : G \to G$, where $f(x) = xe$, is an involutive mapping, $f = f^{-1}$.*

Theorem 2. *Let $(G,\cdot)$ be a topological CM-quasigroup $(G,\cdot)$ with an $(1,2)$-identity e and $x^2 = e$. If P is an open compact subset* *such that $e \in P$, then P contains an open compact CM-subquasigroup $(Q, \cdot)$ with an $(1,2)$-identity of $(G, \cdot)$.*

In the context of topological groups an analogous result appears in the work of Pontrjagin ($[3], Theorem 16$), whereas the case of left medial topological quasigroups was treated by Chiriac in [4].

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A note on the monomial characters of a wreath product of groups

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We recall that a finite group G is monomial, if any irreducible character χ of G is induced by a linear character λ of a subgroup H of G. More generally, a finite group G is quasi monomial, if for any irreducible character χ of G, there exist a subgroup H of G and a linear character λ of H such that $\lambda^G = d\chi$ for a positive integer d.

The notion of *almost monomial groups*, which is a loose generalization of (quasi) monomial groups, was introduced by F. Nicolae in a recent paper [2], in connection with the study of Artin L-functions associated to a Galois extension $K/\mathbb{Q}$. A finite group G is called *almost monomial*, if for any two irreducible characters $\chi \neq \phi$ of G, there exist a subgroup $H \leqslant G$ and a linear character λ of H, such that χ is a constituent of the induced character λ^G and ϕ is not.

A key ingredient in the proof of Dade's theorem ([1,Theorem 9.7]), i.e. any solvable group is isomorphic to subgroup of some monomial group, is the following result: If A is a monomial group and C is cyclic of prime order $p > 0$, then the wreath product $W = A \wr C$ is monomial. The main purpose of this note is to generalize this result in the framework of quasi monomial and almost monomial groups: We prove that if A is quasi monomial (or almost monomial) and C is cyclic of order $p > 0$, then $W = A \wr C$ is quasi monomial (respectively almost monomial) if certain technical conditions hold.

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Relatively hereditary radical classes of rings

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A (Kurosh-Amitsur) *radical class* of rings is a non-empty homomorphically closed class $\mathcal{R}$ such that for each ring A,
$\sum\{I \triangleleft A : I \in \mathcal{R}\} \in \mathcal{R}$ and
$\mathcal{R}(A/\mathcal{R}(A)) = 0$.

Examples *Jacobson* radical class consists of all quasiregular rings $((\forall a)(\exists b)a + b + ab = 0)$.
Nil radical class is the class of nil rings $((\forall a)(\exists n)a^n = 0)$.
Idempotent radical class E consists of all rings A for which $A^2 = A$.

The first two are *hereditary*: they contain all ideals of all their members. Considerable work has been done on radical classes $\mathcal{R}$ with even stronger closure properties, e.g. being *left-hereditary* ($A \in \mathcal{R} \Rightarrow$ $\mathcal{R}$ contains all left ideals of A) and *strongly hereditary* ($A \in \mathcal{R}$ implies $\mathcal{R}$ contains all subrings of A).

There has been little attention to radical classes which are hereditary for some ideals but not (necessarily) all ideals. For instance nothing seems to be known about radical classes which are hereditary for *maximal* ideals, or for *prime* ideals.

A radical class $\mathcal{R}$ is hereditary if and only if $\mathcal{R}(I) = I \cap \mathcal{R}(A)$ for every ideal I of every ring A; [1], [2], p.46. We have some generalizations of this. For a non-empty class $\mathcal{C}$ of non-zero rings which is hereditary for non-zero ideals, we call an ideal I of a ring A a $\mathcal{C}$-*ideal* if $A/I \in \mathcal{C}$. For example when $\mathcal{C}$ is the class of simple (resp.

prime, resp. non-zero finite) rings, the $\mathcal{C}$-ideals are the maximal (resp. prime, resp. finite index) ideals.

For any class $\mathcal{C}$ as described, a radical class $\mathcal{R}$ is hereditary for $\mathcal{C}$-ideals if and only if $\mathcal{R}(I) = I \cap \mathcal{R}(A)$ for every $\mathcal{C}$-ideal I of every ring A.

This result is not definitive: the *essential* ideals satisfy its conclusion but these are not the $\mathcal{C}$-ideals for any $\mathcal{C}$.

If $\mathcal{C}$ is also closed under non-zero homomorphic images, then for every class $\mathcal{M}$ which is hereditary for $\mathcal{C}$-ideals, so is its lower radical class. (This generalizes a well known property of hereditary classes. [3], [2], p.49.)

The class $\mathcal{D}^*$ of rings with divisible additive groups is a radical class which is not hereditary (for all ideals) but is hereditary for maximal and for prime idels and vacuously for ideals of finite index. Hence for every hereditary radical class $\mathcal{R}$ $\mathcal{R}\cap\mathcal{D}^*$ has these relatively hereditary properties and in many cases is not hereditary, though sometimes it is, e.g. when $\mathcal{R}$ is the class of *regular* rings.

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The implicit reducibility in super-intuitionistic logics

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Here are examined logics that are intermediary between classical logic and intuitionistic logic. They are constructed on finit or infinit chains (i.e. linear ordered sets) of the values. It is well-known that a logic is called a chain if the formula $((p \supset q) \vee (p \supset q))$ is true.

The funcition f of the algebra A is called parametrically expressed [1] by means of a system of functions Σ if there exist functions $g_1, h_1, \ldots, g_r, h_r$ which are expressed explicitly via the system Σ using superposition such that the predicate $f(x_1, \ldots, x_n) = x_{n+1}$ is equivalent to the predicate $\exists\, t_1 \exists\, t_2 \ldots \exists\, t_l \big((g_1 = h_1) \& \ldots \& (g_r = h_r)\big)$ on A. If $t_1, t_2, \ldots, t_l$ are absent, then it is called implicit expressibility. The function f is called implicitly reducible to system Σ if there exists such sequence of functions $f_1, f_2, \ldots, f_m$ that $f_m = f$ and f_i is implicitly expressible via $\Sigma \cup \{f_1, f_2, \ldots, f_{i-1}\}$, for each $i = 1, 2, \ldots, m$.

Let $Z_m = \big\langle \{0, r_1, r_2, \ldots, r_{m-2}, 1\}; \Omega \big\rangle$ be a pseudo-Boolean algebra, where $0 < r_1 < r_2 < \ldots < r_{m-2} < 1$, $\Omega = \{\&, \vee, \supset, \neg\}$ and LZ_m denotes the set of valid formulas, i.e. the logic of Z_m. Also, let $\varphi(0) = 0$, $\varphi(r_1) = \varphi(r_2) = r_2$, $\varphi(1) = 1$ and $\psi(0) = 0$, $\psi(r_1) = 1$, $\psi(r_2) = r_2, \psi(1) = 1$ on Z_4.

The system Σ of formulas is called complete by the implicit reducibility in logic LZ_m if each formula is implicitly reducible in LZ_m to Σ.

The criterion of completeness relative to implicit reducibility in LZ_3 has been obtained earlier by the author [2].

Theorem *For any $m = 4, 5, \dots$ in order that the system Σ of formulas could be complete by the implicit reducibility in logic LZ_m it is necessary and sufficient that Σ be complete by the implicit reducibility in logic LZ_3 and be not included in the following two formulas centralizers in algebra Z_4: $\langle \varphi(p) \rangle, \langle \psi(p) \rangle$.*

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Orthogonality of matrix quasigroups

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Suppose K is a commutative finite ring with a unit element, $K^n := K \times K \times \cdots \times K$. Groupoids $(K^n; f)$ and $(K^n; g)$ being defined by

$$f(\bar{x}, \bar{y}) = \bar{x}A + \bar{y}B + \bar{a}, \qquad g(\bar{x}, \bar{y}) = \bar{x}C + \bar{y}D + \bar{c}, \tag{1}$$

where A, B, C, D are square matrices of order n over an m-element ring K and $\bar{a}, \bar{c} \in K^n$, are called *matrix quasigroups*, if A, B, C, D are invertible. Matrix quasigroups $(K^n; f)$ and $(K^n; g)$ having the canonical decompositions

$$f(\bar{x}, \bar{y}) = \bar{x}A + \bar{y}B, \qquad g(\bar{x}, \bar{y}) = \bar{x}C + \bar{y}D \tag{2}$$

are called *unitary matrix quasigroups.* [1]

Two binary operations f and g defined on Q are called orthogonal if a system $\{f(x,y) = a,\ g(x,y) = b\}$ has a unique solution for all $a, b \in Q$.

Theorem 1. *Two matrix quasigroups* $(K^n; f)$ *and* $(K^n; g)$ *with the canonical decompositions* (1) *are orthogonal if and only if* $BA^{-1} - DC^{-1}$ *(or* $CD^{-1} - AB^{-1}$*) is an invertible matrix.*

We give below some corollaries for unitary matrix quasigroups possessing cross inverse properties (CIP). A quasigroup $(Q; \cdot)$ is called: a *middle* CIP *quasigroup*, a *left* CIP *quasigroup*, a *right* CIP *quasigroup* if there exists a transformation α, β, γ such that for all x and y the respective equality holds:

$$\alpha(x) \cdot yx = y, \qquad yx \cdot y = \beta(x), \qquad y \cdot xy = \gamma(x).$$

It was shown in [1] that if unitary matrix quasigroups $(K^n; f, \bar{0})$ and $(K^n; g, \bar{0})$ with canonical decompositions (2) are

1) middle CIP quasigroups, then $B = A^{-1}$, $D = C^{-1}$;
2) left CIP quasigroups, then $B = -A^2$, $D = -C^2$;
3) right CIP quasigroups, then $A = -B^2$, $C = -D^2$.

Corollary 1. *Two unitary matrix middle* CIP *quasigroups* $(K^n; f, \bar{0})$ *and* $(K^n; g, \bar{0})$ *with canonical decompositions* (2) *and the property* $AC = CA$ *are orthogonal if and only if* $C - A$ *and* $C + A$ *are invertible matrices.*

Corollary 2. *Two unitary matrix left* CIP *quasigroups* $(K^n; f, \bar{0})$ *and* $(K^n; g, \bar{0})$ *with canonical decompositions* (2) *are orthogonal if and only if* $C - A$ *is invertible matrix.*

Corollary 3. *Two unitary matrix right* CIP *quasigroups* $(K^n; f, \bar{0})$ *and* $(K^n; g, \bar{0})$ *with canonical decompositions* (2) *are orthogonal if and only if* $D - B$ *is invertible matrix.*

The similar statements can be formulated for matrix quasigroups with inverse properties (IP). Their canonical decompositions one can find in [2].

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On the theory of the generalized symmetry of the geometrical figures regularly weighted by "physical" scalar or oriented tasks

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The theory of symmetry of the real crystal gives rise to new generalizations of classical symmetry: the Shubnikov antisymmetry [1], the multiple antisymmetry of Zamorzaev [2], the Belov's color symmetry [3], the Zamorzaev's P-symmetry [4], the $\overline{P}$-symmetry [5], the W_p-symmetry [6,7], the W_q-symmetry [8-10].

In this paper we discuss the essence of the mixed transformations of a geometric figure, regularly weighted by "physical" scalar or oriented tasks (compare to [11,12]). In accordance with global nature or local nature of the rule for transformation of "indexes", ascribed to each point of the geometric analyzed figure, four types of mixed transformations are obtained. Are determined the conditions in which one mixed transformation is exactly transformation of P-symmetry, or exactly transformation of W_p-symmetry (the broadest generalization of classical symmetry when the "indexes"-qualities have a scalar character). Moreover, when one mixed transformation is exactly transformation of $\overline{P}$-symmetry (Q-symmetry), or exactly transformation of W_q-symmetry (the widest generalization of classical symmetry when the qualities, located in the points of the figure, are homogeneous and with different orientations) . Some properties of the groups of W_p-symmetry and of the groups of W_q-symmetry are studied.

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T-quasigroups with Schröder identity $xy \cdot yx = y$

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We study T-quasigroups with Schröder identity $xy \cdot yx = y$ [1, 2, 3].

Definition. *Quasigroup $(Q, \cdot)$ is a T-quasigroup if and only if there exists an abelian group $(Q, +)$, its automorphisms φ and ψ and a fixed element $a \in Q$ such that $x \cdot y = \varphi x + \psi y + a$ for all $x, y \in Q$ [4].*

A T-quasigroup with the additional condition $\varphi\psi = \psi\varphi$ is medial.

Theorem. *In T-quasigroup $(Q, \cdot)$ of the form $x \cdot y = \varphi x + \psi y$ Schröder identity is true if and only if $\varphi^2 + \psi^2 = 0$, $\varphi\psi + \psi\varphi = \varepsilon$.*

Corollary. *In medial quasigroup $(Q, \cdot)$ of the form $x \cdot y = \varphi x + \psi y$ Schröder identity is true if and only if $\varphi^2 + \psi^2 = 0$, $2\varphi\psi = \varepsilon$.*

Example 1. Suppose we have the group Z_n of residues modulo n. If $\varphi = 3$, $\psi = 1$ then $\varphi^2 + \psi^2 = 9 + 1 = 0 \pmod 5, n = 5$. Further $2\varphi\psi = 2 \cdot 3 \cdot 1 = 6 = \varepsilon = 1 \pmod 5$, $x \cdot y = 3x + y \pmod 5$. Check. $3(3x + 1y) + 1(3y + x) = y \pmod 5$, $9x + 3y + 3y + x = y \pmod 5$, $y = y \pmod 5$.

Example 2. Suppose we have the group Z_n of residues modulo n. If $\varphi = 10$, $\psi = 2$ then $\varphi^2 + \psi^2 = 100 + 4 = 104 = 0 \pmod{13}, n = 13$. Further $2\varphi\psi = 2 \cdot 10 \cdot 2 = 40 = \varepsilon = 1 \pmod{13}$, $x \cdot y = 10x + 2y \pmod{13}$. Check. $10(10x + 2y) + 2(10y + 2x) = y \pmod{13}$, $100x + 20y + 20y + 4x = y \pmod{13}$, $y = y \pmod{13}$.

Example 3. We construct quasigroup $x \circ y = 2657x + 7063y \mod (9721)$ and check that in this quasigroup 3-rd Stein identity is fulfillment : $2657(2657x + 7063y) + 7063(2657y + 7063x) = y \mod (9721)$, $7059649x + 8766391y + 8766391y + 9885969x = y \mod (9721)$, $56945618x + 37532782y = y \mod (9721)$, $y = y \mod (9721)$.

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On families of subspaces of pointwise countable type

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In [2, 3] the general concept of a metrizable family of subsets of a topological space was introduced and applied to the problem of selections of multivalued mappings. Then in [4] in the similar context the concept of a family of subspaces with first axiom of countability was introduced and studied.

Here the families of subspaces of $\mathcal{Q}$-pointwise countable type are defined and considered. Pointwise countable type spaces were defined by A. V. Arhangelskii in [1].

The property $\mathcal{Q}$ is of a compact type. For example it can be the property k of being a compact space, or the property s of being a countably compact space.

The class of these families is larger than the class of $\mathcal{Q}$-metrizable families of subspaces of a topological space, namely:

Theorem *Assume that a family $\mathcal{A}$ of subsets of a space X is $\mathcal{Q}$-metrizable by the pseudometric d and $\mathcal{Q} \in \{k, s\}$. Then $\mathcal{A}$ is a family of the $\mathcal{Q}$-pointwise countable type.*

The following result is obtained:

Proposition. *Let $i \in \{2, 3, 3.5\}$, a continuous image of a space with property $\mathcal{Q}$ be a space with property $\mathcal{Q}$, $\mathcal{A}$ be a family of subspaces of a T_i-space X and $X = \cup\mathcal{A}$. The following assertion are equivalent:*

1. $\mathcal{A}$ is a family of subspaces of $\mathcal{Q}$-pointwise countable type.

2. There exist a T_i-space Z and an open continuous mapping $f : Z \longrightarrow X$ of the space Z onto the space X such that $\{f^{-1}(L) : L \in \mathcal{A}\}$ is a family of subspaces of Z of $\mathcal{Q}$-pointwise countable type.

3. There exist a T_i-space Z, a continuous pseudometric d on Z and an open continuous mapping $f : Z \longrightarrow X$ of the space Z onto the space X such that $\{f^{-1}(L) : L \in \mathcal{A}\}$ is $\mathcal{Q}$-metrizable by the pseudometric d.

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On recursively differentiable n–quasigroups

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The notions of recursive derivative and recursively differentiable quasigroup have been introduced in [1], where the authors investigate the parameters of MDS-codes, i.e. codes of length n on a set (alphabet) of q elements, with q^k codewords and minimum Hamming distance d, such that $k = n - d + 1$ (which is the maximum value for k). At present it is an open problem to characterize all possible values of the parameters n, k, q, d for which there exist MDS-codes. It is known, in particular, that for $k = 2$ there exist MDS-codes on every alphabet Q, which cardinal is a power of a prime.

A special class of MDS-codes are those which cheek functions are recursive derivatives of a k-ary quasigroup, called *recursive MDS-codes*. The existence of such codes depends on the order of recursive differentiability of the defining quasigroup.

We investigate the recursive differentiability of n-quasigroups, in particular of n-groups (Q, A), where $A(x_1, ..., x_n) = x_1 \cdot ... \cdot x_n$, for every $x_1, ..., x_n \in Q$, and $(Q, \cdot)$ is a finite abelian binary group. Necessary and sufficient conditions of recursive r-differentiability of such n-groups are given ($r \geq 1$). This result is a generalization of the criteria of recursive r-differentiability of a binary abelian group, qiven in [2]. In particular, we prove in the present work that there exist recursively 1-differentiable n-quasigroups of any odd order $q \geq 3$, for $\forall n \geq 2$.

Theorem. *Let $(Q, \cdot)$ be a finite binary abelian group, $n \geq 2$ and $r \geq 1$ be two natural numbers. The n-group (Q, B), where $B(x_1, ..., x_n) = x_1 \cdot ... \cdot x_n$, for every $x_1, ..., x_n \in Q$, is recursively r-differentiable if and only if, for every $i = 1, ..., n$ and every $k = 1, ..., r$, the mappings $x \to x^{s_i^k}$ are bijections in $(Q, \cdot)$, where the sequences $(s_i^k)_{k \geq 0}$, $i = 1, ..., n$, are defined (recursively) as follows:*

1. *$k = 0:\ s_1^0 = ... = s_n^0 = 1;$*
2. *$1 \leq k < n$:*
   $s_i^k = s_i^0 + ... + s_i^{k-1}$, for $\forall i = 1, ..., k;$
   $s_i^k = 1 + s_i^0 + ... + s_i^{k-1}$, for $\forall i = k+1, ..., n;$
3. *$k \geq n:\ s_i^k = s_i^{k-n} + ... + s_i^{k-1},\ \forall i = 1, ..., n.$*

Corollary 1. *There exist recursively 1-differentiable n-quasigroups of any odd order $q \geq 3$, for every $n \geq 2$.*

Corollary 2. [2] *A finite abelian group $(Q, \cdot)$ is recursively r-differentiable if and only if, for every $i = 1, 2$ and every $k = 1, .., r$, the mappings $x \to x^{s_i^k}$ are bijections in $(Q, \cdot)$, where the sequences $(s_1^k)_{k\geq 0}$ and $(s_2^k)_{k\geq 0}$ are defined as follows: 1. $s_1^0 = s_2^0 = 1$; 2. $s_1^1 = s_1^0 = 1,\ s_2^1 = 1 + s_2^0 = 2$; 3. $s_i^k = s_i^{k-2} + s_i^{k-1},\ \ i = 1, 2$, for any $k \geq 2$.*

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Matrix quasigroups with invertibility property

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Let K be a commutative ring with a unit element and $K^n := K \times \ldots \times K$. The groupoid $(K^n; f)$ being defined by

$$f(\bar{x}, \bar{y}) := \bar{x}A + \bar{y}B + \bar{a},$$

where A, $B \in M_n(K)$ and $\bar{a} \in K^n$, is called *matrix quasigroup over the ring K* if the matrix A, B are invertible.

Each matrix quasigroup is central. Each central quasigroup being isotopic to an elementary abelian group is isomorphic to a matrix quasigroup.

A quasigroup $(Q;\cdot)$ is called: a *left IP quasigroup*, a *right IP quasigroup*, a middle *IP quasigroup*, if there respectively exists a transformation λ, ρ, μ such that for all x and y the respective equality holds:

$$\lambda(x)\cdot xy = y; \qquad yx\cdot\rho(x) = y; \qquad xy = \mu(yx).$$

A complete description of quasigroups with invertibility properties according to equalities of sets of their translations has been given in [1] and have been obtained 9 varieties: left, right and middle quasigroups with inverse property (IP), cross inverse property (CIP) and mirror property. They are distributed over 3 parastrophic orbits. In the class of matrix quasigroups, it is sufficient to consider two parastrophic orbits since all mirror quasigroups are IP quasigroups. Classification of matrix IP and CIP quasigroups was considered in the work [2]. In particular, for the matrix IP quasigroup is obtained the following assertion.

Theorem[2]. *Each matrix IP quasigroup over the ring K has the form:*

middle IP	$f(\bar{x},\bar{y}) = \bar{x}A + \bar{y}AC + \bar{a}$
left IP	$f(\bar{x},\bar{y}) = \bar{x}A + \bar{y}C + \bar{a}$
right IP	$f(\bar{x},\bar{y}) = \bar{x}C + \bar{y}A + \bar{a}$
left-middle IP	$f(\bar{x},\bar{y}) = \bar{x}C_1C_2 + \bar{y}C_1 + \bar{a}$
right-middle IP	$f(\bar{x},\bar{y}) = \bar{x}C_1 + \bar{y}C_1C_2 + \bar{a}$
left-right IP	$f(\bar{x},\bar{y}) = \bar{x}C_1 + \bar{y}C_2 + \bar{a}$
left-right-middle sided IP	$f(\bar{x},\bar{y}) = \bar{x}C_1 + \bar{y}C_2 + \bar{a},\ C_1C_2 = C_2C_1$

where the matrix A is invertible and $C^2 = C_1^2 = C_2^2 = E$; $\bar{a}\in K^n$.

Orthogonality of matrix quasigroups, in particular CIP quasigroups, is considered in [3].

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Some universal properties of n-IP-loops

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In the paper we define the property of universality of n-loop and we study this property in n-IP-loop, n-IP-loop Moufang and n-TS-loop.

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On normal 3-isohedral spherical tilings for group series $n\times$

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A tiling W of the sphere with disks is called 3-isohedral with respect to an isometry group G if G maps W onto itself and the tiles of W fall into exactly 3 transitivity classes under the group G. Recently we have obtained all the fundamental 3-isohedral tilings of the sphere with disks for group series $n\times$, $n = 1, 2, \ldots$, which comprise 293 series of Delone classes.

In [1] B. Grünbaum and G. C. Shephard distinguish the so-called normal tilings with the additional restriction that the intersection of any set of tiles is a connected (possibly empty) set. These intersections define edges and vertices of the tiling. One more restriction is that each edge of the tiling has two endpoints which are vertices of the tiling.

We choose tilings satisfying the normality conditions among all fundamental 3-isohedral tilings of the sphere with disks for group series $n\times$. As a result there are 92 series of Delone classes of normal fundamental 3-isohedral tilings of the sphere for group series $n\times$. Earlier all the normal fundamental tilings of the sphere for group series $*nn$, nn, $*22n$, and $n*$ were listed in [2].

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7. Computer Science

Study of the convergence of computer calculations

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The problem of reliability of computer calculations is one of the fundamental problems of Computer Science, as it lies at the intersection of applied mathematics on the one hand, and physical and technical limitations of computer technology on the other.

Physical limitations of the amount of memory allocated by computer technology for storing and processing numbers impose fundamental limitations on the possibilities and logic of organizing computer calculations, which also requires the use of rather specific and non-obvious mathematical research methods.

It was considered the mathematical object that was given by Z.-M. Muller and co-authors in the fundamental work [1], devoted to the problem of applied computer calculations. This is a nonlinear inhomogeneous second-order recurrence equation, which we will call the Muller's recurrent sequence:

$$\begin{cases} u(n) = 111 - \dfrac{1130}{u(n-1)} + \dfrac{300}{u(n-1)\cdot u(n-2)}, \\ \hfill u(0) = 2, u(1) = -4. \end{cases}$$

As a result of the research, a mathematical justification of the problem of unstable initial values was built and a formula for checking the initial values for the considered sequence was derived.

It was also proved the convergence of Muller's recurrent sequence using the method of mathematical induction, and that although the investigated sequence coincides, a significant error occurs during computer calculations of the members of the sequence.

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Platform for Digitization of Heterogeneous Documents

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The digitization platform is a web/desktop application written in Python and Javascript, which integrates the processing stages of heterogeneous documents into a digitization cycle, consisting of the following main steps: uploading images or/and PDF files, image preprocessing, optical recognition of characters in the image, checking and editing of recognized text, transliteration of text after checking the recognized text, checking and editing the transliterated text and finally saving the results to the database and/or downloading them. These steps, in turn, branch into a list of sub-steps, which we will detail below.

It is worth mentioning the technical peculiarities of implementing this platform, namely:

- some of the data operations, e.g. some image processing methods, are available as JavaScript libraries and are therefore executed in the front-end;

- some services, mainly heterogeneous content recognition, are called from the network via their own APIs, which essentially means that there are multiple backends.

Most of the steps contain submenus, through which the user can choose one of the proposed options (tools). For example, preprocessing of the initial image can be done using FineReader, Open CV, ScanTailor or Gimp.

One of the most important stages is OCR, this step is applied to the preprocessed document and starts with classifying the heterogeneous content and fragmenting the document into homogeneous components. For the time being the following types of sub-items are provided: image, text, musical notes, mathematical formulas, chemical formulas and structures, chess diagrams. A user-friendly interface is developed in the form of a dialog, which works via API on the user side (frontend) and on the server side (backend): the image file can be uploaded, the heterogeneous content can be identified, it can be split into fragments, it can be analysed and recognised, the resulting file can be viewed with the possibility of saving it.

Acknowledgments. The platform was developed as a part of project 20.80009.5007.22 “Intelligent information systems for solving ill-structured problems, processing knowledge and big data”.

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New algorithms and their software implementations for some combinatorial games

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Combinatorial game theory is a mathematical theory that studies two-person games in which there is a position at any given time that players alternately alter (according to the rules) to achieve a win. This theory does not study games related to chance, only games in which positions and all known moves are known to both players [1], [2].

The application of combinatorics game theory to a particular position in a game (conflict) consists of the determination of the optimal sequence of motions for players to the end of game, that is by the determination of optimal motion in every position [3].

For many combinatorial games of perfect information (for example, Go, Nim and TakTix) it is known that there are algorithms that determine whether or not the player moving first has a "forced win" from a given starting position. So we can conclude that such an algorithm decides the game. To such algorithms, it is possible to take a decision tree (schematic presentation of the problem of making a decision), greedy algorithm (basic problem to take away as possible any more chips from the field), symmetric algorithm (an imitation of the motion of rival that is on other part of the playing field) and others. One of the combinatorial games is TakTix, developed by Pete

Hein. Chips (or coins, pebbles) are laid out on a board of size $n \times m$ or $n \times n$. At the beginning of the game, players determine the maximum and the minimum number of chips that can be taken in one move. Each player is allowed to take any horizontal or vertical sequence of consecutive chips. TacTix is played with the misre version, meaning that the player that takes the last chips loses, or wins in the non-misre version.

There exist winning strategies for players (for non-misere games):

- For the First Player : If N is odd, then take the centerpiece and copy every move your opponent makes symmetrically. Eventually, you will take the last piece and will win.

- For the Second Player - If N is even, then copy your opponent's moves symmetrically. Eventually, you will take the last piece and will win. The proposed algorithm for the TacTix game is more optimal than existing algorithms. The application of the algorithm does not affect the computer implementation of the game, but the number of moves to win is more optimal. The computer implementation includes the ability to play not only with the computer but also with another player.

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Some solutions for applications of Intelligent information systems in medicine

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At the 2015 Davos forum, among the most impressive achievements predicted for 2025 are regular internet access for 90

However, it is clearly that the role of AI is becoming crucial for many fields of activity, including creative ones. One topic of particular importance is the use of AI in healthcare, and the COVID 19 pandemic has confirmed this. According to officials at Moderna, the rapid development of the vaccine was largely made possible by the application of artificial intelligence. More and more widespread are AI-based approaches in medical image processing (in particular - in cancer detection), remote consultations, diagnosis. Such approaches have also been applied by researchers at the V.Andrunachievici Institute of Mathematics and Computer Science in the development of intelligent information systems for medicine. In what follows, we will briefly review them.

Solutions in the SonaRes system were developed to provide the possibility to use the experience of experts gathered in the knowledge base of the system, to consult the annotated images, similar to the ones examined and guide the examination process, adapting to the different level of experience of the doctor. An essential feature is assistance in the preparation of the report ensuring compliance with a single standard. It is also important to prevent possible errors in the examination process (such as omitting some important aspects or characteristics in the examination, admitting inaccuracy in the formulation of the conclusion, etc.) SonaRes provides annotated images, help, illustrations and explanations in difficult cases especially important for inexperienced young people. offers the possibility of being used in training.

Information System Support for Cerebrovascular Accident prophylaxis. The project within the State Program Systemogenesis of risk factors, optimization of the health service, sustainable evaluation and mathematical modeling of stroke was focused on the development of personalized mathematical prediction models (including for small volume passive samples). The computer system STROKE.MD was developed to support the collection and processing of data and the development of personalized prediction models. Machine learning based methods were used to develop the predictive models, especially those incorporated in the WEKA system.

AI Based Multilayered Approach for Management of Mass Casualty Situations. The project jointly developed with researchers from Germany, Romania, Croatia and the USA aimed to develop software for mobile devices with a simple user interface (via voice recording) to collect and organize primary medical data of victims and create DSS systems for the efficient placement and transport of victims, providing guidance for rapid transport based on innovative AI inference frameworks and transport systems but also for the medical evaluation of stabilized victims (based on ultrasound characteristics) during transport or in clinical conditions.

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On constructing a linguistic model of the Romanian language using geographically referenced dialect data

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In this paper the peculiarities of the construction of linguistic models on the basis of the Romanian dialects of the Republic of Moldova are considered. The source of the data is a six-volume collection of phonetic records of Romanian dialect speech from 240 localities [1]. The aim of the work is to build a neural model that correlates pronunciation peculiarities with the location. Data preparation is currently underway. The finished model can be used to tune manually constructed isoglosses, to identify fragments of phonetic records, to calculate the degree of similarity of dialects, etc.

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How to build up an Artificial Neural Network in C

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From a mathematical point of view, Artificial Neural Networks (ANNs) are a class of artificial intelligence algorithms aimed to

- find patterns in a large data base,
- approximate differential equations,
- reconstruct the functions from scatter data etc.

The main feature of an ANN model is the ability to *think*: to interpret data accounting for context. Nowadays, there is a multitude of software tools that implement ANNs, but most of them hide the mathematical background. In this paper, we discuss the mathematical tools needed to build up an ANN software using C-language.

Artificial intelligence and parallel programming for processing of big data

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Intelligent information systems, represented by artificial intelligence (AI), are most suitable for solving unstructured problems.

They are based on the identification of features, on the basis of which are drawn certain conclusions, and is planned a further strategy for problem solving. A human can possess a lot of incredibly important and valuable information. In turn a computer can bring together the professional experience of many minds of different generations, scaling the internal database of tasks through self-learning. Depending on the assigned task, is efficient to use parallel programming to process big data. Due to the more efficient use of computer capabilities, the data processing speed increases at least twice[1], which means that the execution time is also proportionally reduced. But this approach also requires a different logic for organizing the functions inside the program. It is an unchangeable fact that any data need intellectual analysis that may be impossible to provide for some reasons. As noted earlier, human capacity is limited compared to computers, and the ever-increasing amount of information being processed makes it necessary to think about other approaches that would fully cover the needs that arise. By parallelizing some processes, it is possible to speed up the training of artificial intelligence. It may also affect the possible search for new theorems or patterns not previously studied by humanity.

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Using VPN to secure data

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Rapid technological changes in today's society and the intense development of science and technology impose high demands on the digital skills of specialists in various fields of activity.Among the basic digital literacy, information security is also included.

Virtual Private Network (VPN) is a security tool, essential for privacy and data protection, which is routing data through strong online encryption.

Without a VPN, your Internet traffic can be easily intercepted and viewed by others, including your browsing history, downloaded files, bank details, passwords, etc. Using a VPN includes data privacy and security, device protection when using public WiFi networks, removing geo-restrictions, remote access to the corporate network in order to avoid the risk of data theft, etc

Just like with many other online privacy tools, the actual level of VPN security depends on the service used. Basically, VPN protects online life by encrypting Internet traffic and hiding your real IP address [1].

A secure VPN uses strong data encryption, no data tracking, no IP address data leaks, no DNS (Domain Name System) leaks, multi-factor authentication, etc.

Security threats are present with cheap or free VPNs. This type of network has both advantages and disadvantages. Advantages: accessing restricted entertainment content, accessing social media sites or news services, maintaining anonymity while private web browsing, etc.

Disadvantages: Free VPNs save money but don't guarantee safety, can sometimes slow down your online speed, and don't inherently protect your privacy or give you anonymity, etc. [2, 3].

This article highlights the best VPNs for data protection, as well as if data can be stolen, and last but not least, whether a VPN connection can be hacked. It provides a unique and in-depth look at the major business challenges and threats that are introduced when an institution's network is connected to the public Internet.

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Marker-Based Augmented Reality approach used in learning geometry

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Augmented reality makes impossible things possible and the possibilities offered by it in education [1] are still unsettled, and in development. In Republic of Moldova the trend of implementing AR in education are very new, thus, through our research we intend to bring our contribution in implementing these new technologies in the teaching process. We develop an AR tool that enhance the learning effect by tracking 2D artefacts that trigger the visualization of the 3D geometry objects using marker-based Augmented Reality approach [2].

We designed 30 types of MB-AR artefacts, when scanned by mobile devices camera they trigger one of augmented experiences, such us: 3D objects, video content, audio content, text, formulae, even virtual tutor. Using marker-based augmented developed system learners can interact with the 3D information, objects and events in a natural way.

Additional, pupils can interact with artefacts to change the position, size and color of 3D objects. Moreover, by interacting with virtual tutor pupils can see the superimposed digital content that explains and demonstrate the basic theorems. The tool is created by using Unity platform with Vuforia database.

MB-AR tool will deliver a positive impact by keeping pupils' high engagement and by enhancing their learning abilities like problem-solving, collaboration, imaginative thinking and spatial imagination.

In the following we will take learning process to the next level by helping pupils not just to see, but also experience and practice, changing the abstract concept into the tangible one, also will be done work on diversifying the scenarios, by highlighting learning styles, adding adaptive tests depending on learning styles. Additionally, it will be useful for pupils with physical inactivity, mental health, dementia, autism problems.

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Application of Machine Learning in the Research of an Unknown Text

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For almost 34 years, WRI has been developing the Wolfram Language, and Mathematica System, as a major computation and programming environment for creative categories of people [S. Wolfram; C. Hastings and others]. The Wolfram Language permits us to apply the most advanced computations and knowledge in areas that are extremely far from programming, numeric, symbolic, and technical computations.

We present an illustration how the Wolfram Language, and Mathematica System, can be used to investigate poetry and prose, translations of literary works to other languages, evaluation of translation quality, discovering the author of unknown text, and highlighting plagiarism.

We begin by obtaining some numerical characteristics of works in different original languages, and their translations into others, e.g., from Romanian to English, French, and Russian. A good translation must preserve most characteristics of the original works. Is this so in practice? To make more objective conclusions, some graphical, image, and sound perspectives are presented. We can also use some advanced mathematical tools such as interpolation, and curve fitting. Based on interpolation-functions (or fit-functions) that correspond to original works and their translations, we can evaluate good translation-works as ones for which interpolation functions for translated text (fit-functions) differ insignificantly from the interpolation-functions (fit-functions) of the original works and are close one to other.

We continue with the application of Machine Learning to train a function that may recognize poetry and prose texts, which may find text's author, too. Should we train a function for every language, or is it enough to train one function for all languages? May a trained function have a "polyglot" feature? If the trained author-function "understand" more than one language, may it be applied to evaluate good work-translation? Should the trained author-function understand who is the original language author of translated work?

We present answers to the above questions and highlight a series of other interesting subjects which arise in this context. The final discussion and conclusions are a good starting point to an interesting area of research: computational recognition of the original language author for a translated work.

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